LEARNING AND UNCERTAINTY QUANTIFICATION FROM COMPLEX SIMULATIONS

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Division of Digital Sciences and Technologies, Department of Applied Mathematics

April 28, 2026

LEARNING AND UNCERTAINTY QUANTIFICATION FROM/WITH SDE MODELS AND COMPLEX SIMULATIONS

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April 28, 2026
UCC Cork, Ireland

IFPEN AT A GLANCE

Public sector
R&I institution

Training
center

Industrial
Group

INTERNATIONAL SCOPE in the field of
ENERGY, MOBILITY and **THE ENVIRONMENT**

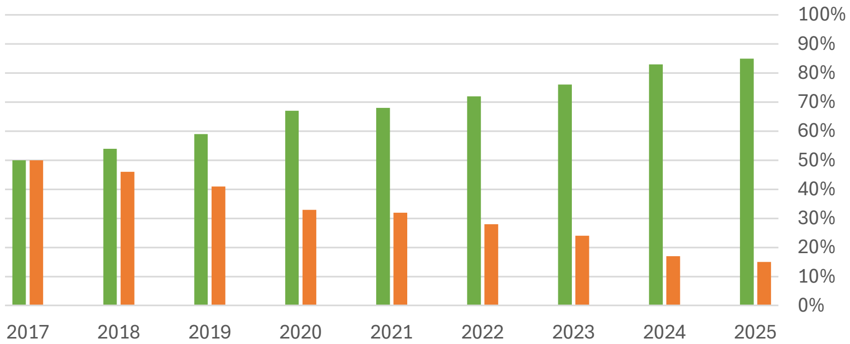
1,523 employees
incl. **1,074 R&I engineers & technicians**

€117,6 M Budget allocation in 2025
€162,1 M Own resources in 2025

Research & Innovation activities

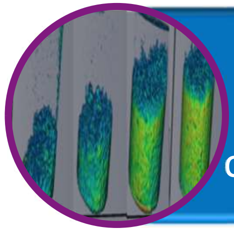
Green technologies

Responsible & profitable Oil & Gas

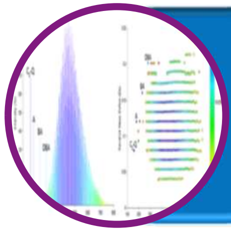


FUNDAMENTAL RESEARCH AROUND 9 SCIENTIFIC CHALLENGES

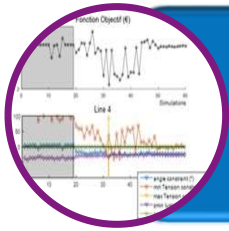
From the understanding of physical phenomena to the evaluation of a complete system, via numerical modeling of these phenomena



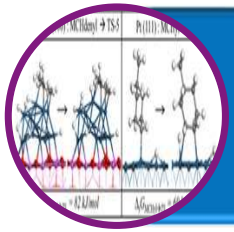
Material and fluid characterization



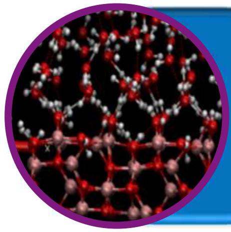
Massive data flows



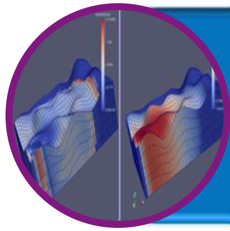
Command and optimization



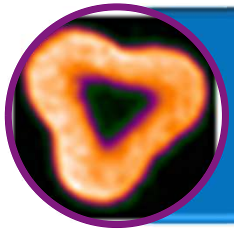
Reaction mechanisms



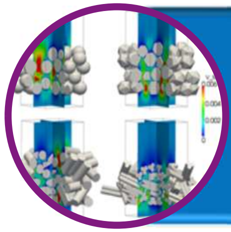
Descriptors



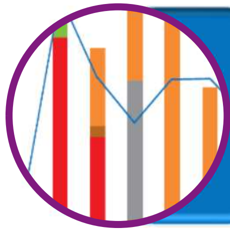
Code performance



Containment effect



Modeling of coupled phenomena



Economic and environmental evaluations

OUR FIELDS OF COMPETENCE

Earth sciences
- Geology – Sedimentology
- Geochemistry
- Geostatistics – Geological modeling
- Geomechanics
- Petrophysics and transfers in porous media

Chemical Sciences
- Catalysis and reaction kinetics
- Organic and mineral synthesis
- Separation and adsorption techniques
- Theoretical chemistry

Analysis and Characterization
- Chemical analysis
- Structural analysis and imaging
- Mechanical testing
- Microfluidics
- High throughput experimentation (HTE)

Physical Sciences
- Transfer and transport physics
- Rheology and behavior of materials
- Thermodynamics / Molecular modeling

Physical chemistry
- Complex fluids, colloids and condensed matter
- Surface, interface and materials science
- Electrochemistry and corrosion

Biosciences and Biotechnology
- Microbiology
- Genomics
- Biocatalysis
- Fermentation

Engineering Sciences
- Solid mechanics
- Fluid mechanics
- Chemical and process engineering
- Combustion and engine technologies
- Electrical and electronic engineering
- Automation and control systems
- Systems modeling and simulation

Mathematics And Computer Sciences
- Numerical Methods, UQ and optimization
- Signal processing – Data science
- Meshing and visualization
- Software design
- Real-time systems
- High performance computing
- Bio-informatics

Economics
- Microeconomics and econometrics
- Macroeconomics
- Economic modeling
- Forecasting and scenario modeling
- Technical and economic evaluation
- Environmental impact evaluation and life-cycle assessment

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- Wind field generation
  - Stochastic wind from SDE?
  - Non-linear stochastic frequency-domain simulators?
  - Towards Machine-learned non-stationary wind generator (PhD)

} Part I

- UQ of complex simulators, the Black-box framework
- Active learning and Poincaré-Malliavin-Nualart inequalities

} Part II

- Metamodelling of complex simulator with functional inputs, bridging Gaussian processes and SDE

} Opening

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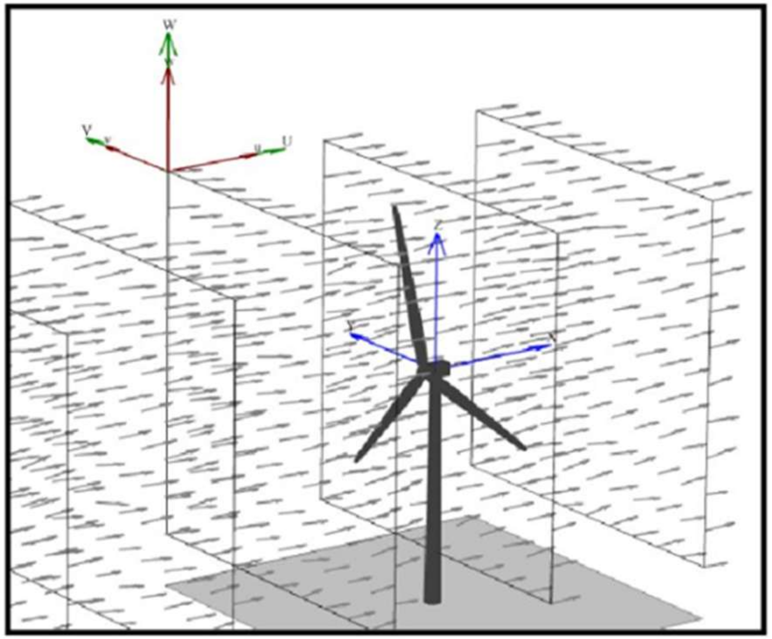
WIND VELOCITY AS A STATIONARY GAUSSIAN PROCESS



The rotor-plane wind field is modelled as a **vector stochastic process**:

$$\mathbf{U}(x,y,t) = \big(u(x,y,t),\ v(x,y,t),\ w(x,y,t)\big)$$

Under the **stationary hypothesis**, all statistical moments are constant:

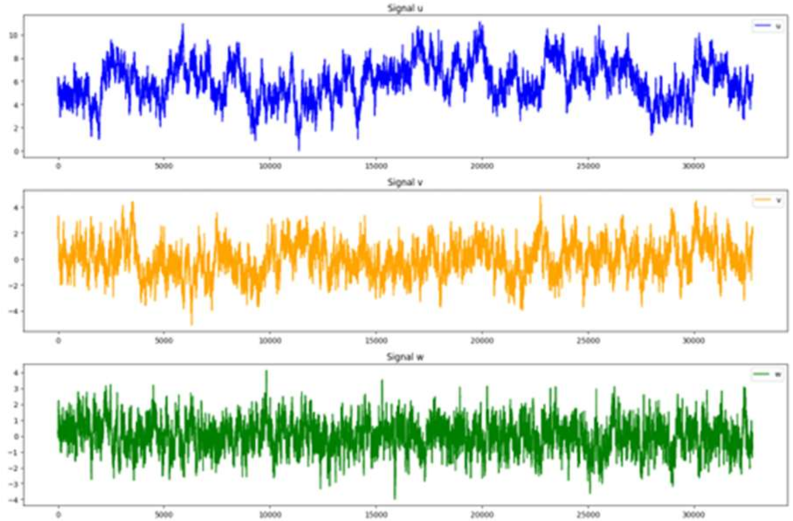
- mean U_{hub},
- variance σ_u^2,
- Kaimal one-sided PSD (Power Spectral Density)

$$S_u(f) = \sigma_u^2 \frac{4L_u/U_{\text{hub}}}{(1+6fL_u/U_{\text{hub}})^{5/3}}$$

IEC 61400-1 Ed.3 Eq.(A.10); Kaimal et al. (1972)

DIRECT PIPELINE USING TURBSIM LIKE GENERATOR

1. Define PSD $S_u(f),\ S_v(f),\ S_w(f)$
2. Spatial coherence $\mathrm{Coh}(\Delta r, f) = e^{-k\,f\,\Delta r/U}$
3. Cross-PSD matrix $S_{ij} = \sqrt{S_i S_j} \cdot \mathrm{Coh}_{ij}$
4. Cholesky factor $\mathbf{S} = \mathbf{H}\mathbf{H}^T \to$ random phases
5. Inverse FFT $u(x, y, t)$ in time domain

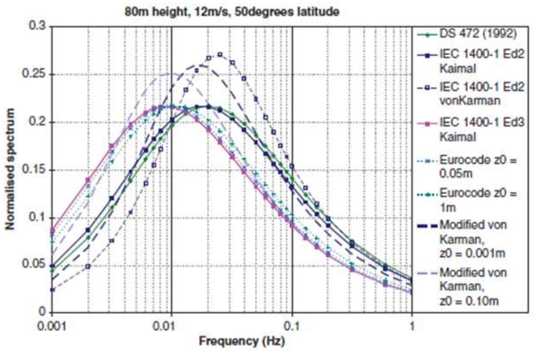


$u(x_0, y_0, t),\ v(x_0, y_0, t),\ w(x_0, y_0, t)$ for (x_0, y_0) at the hub center.

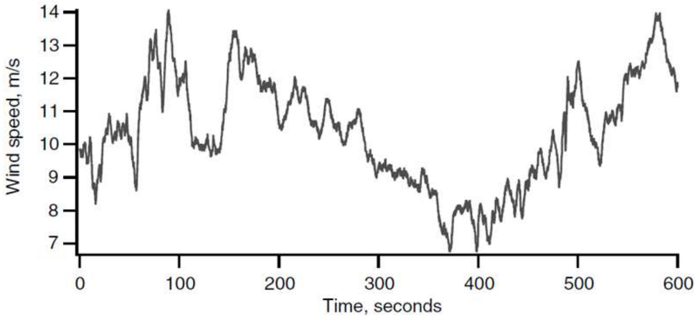
STOCHASTIC WIND FROM EDS?

Jean-Lou PFISTER (IFPEN)

- Usual engineering approach: define parametrized temporal and spatial correlations from a spectrum, then compute the vector-valued 3D wind speed timeseries[1,2]
  - Pros: temporal/spatial correlations are experimentally measurable
  - Cons: timeseries generated once and for all (memory/cpu issues for long timeseries)



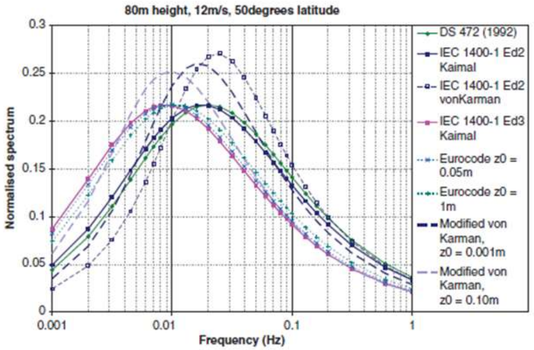
Stochastic wind generator
(TurbSim, pyConTurb, Hipersim…)



[1] Shinozuka & Deotadis, 1991 ; [2] Veers, 1988

STOCHASTIC WIND FROM SDE?

Jean-Lou PFISTER (IFPEN)



Stochastic wind generator
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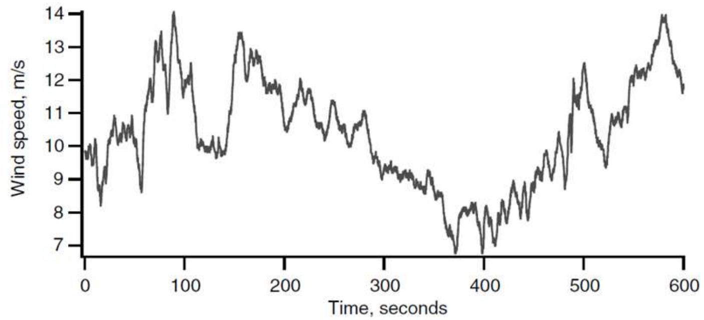

- Alternative approach(?): define parametrized temporal and spatial correlations, find a SDE whose solutions would match these statistics, and solve it on-the-fly
  - Could solve computational issues
  - Possibility to add richer dynamics (transients, extreme events, etc.)
  - Easier to compute derivative based sensitivities (for instance impact of TI on output loads)

NON-LINEAR STOCHASTIC FREQUENCY-DOMAIN SIMULATORS?

Jean-Lou PFISTER (IFPEN)

- Usual engineering approach:
  - Frequency-domain
    - Define parametrized temporal and spatial correlations from a spectrum,
    - then compute the vector-valued 3D wind speed timeseries,
  - **Time-domain**
    - **Solve a non-linear aero-servo-hydro-elastic wind turbine time-domain simulator,**
  - Frequency-domain
    - Extract spectral properties (vibration frequencies & amplitudes)

- **Can we stay in the frequency domain?**
  - May speed-up dramatically the calculations – e.g. assuming an almost-periodic behaviour of the turbine (non-linear periodic motion + random linear fluctuations)
  - May enable an efficient calculation of the sensitivities of outputs wrt input wind temporal/spatial correlations

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- Wind field generation
  - Stochastic wind from SDE?
  - Non-linear stochastic frequency-domain simulators?
  - Towards Machine-learned **non-stationary** wind generator (PhD)
- SDE for molecular dynamics (PhD)
- UQ of complex simulators, the Black-box framework
- Active learning and Poincaré-Malliavin-Nualart inequalities
- Metamodelling of complex simulator with functional inputs, bridging Gaussian processes and SDE

SOURCE OF NON-STATIONARITY

PhD Simon QUERNE (Nov. 2024 – Nov. 2027)
Supervision: Laurent DUMAS (UVSQ), Charles Tillier (UVSQ), **Jean-François LECOMTE (IFPEN)**

Slowly varying mean

Synoptic-scale pressure gradients change U_{hub} on minute-to-hour time scales. Deterministic trend or slow AR process.

Wind veer

Direction changes with height introduce cross-correlation between u and v. Ignored in the IEC simplified model.

Intermittent turbulence

ABL transitions (stable $\rightarrow$ convective) modify σ_u and L_u non-smoothly. Regime-switching Markov models apply.

Gusts and frontal passages

Deterministic transients (EOG, EWS) superimposed on background turbulence. Duration $\mathcal{O}(10\,\text{s})$.

Wind shear variations

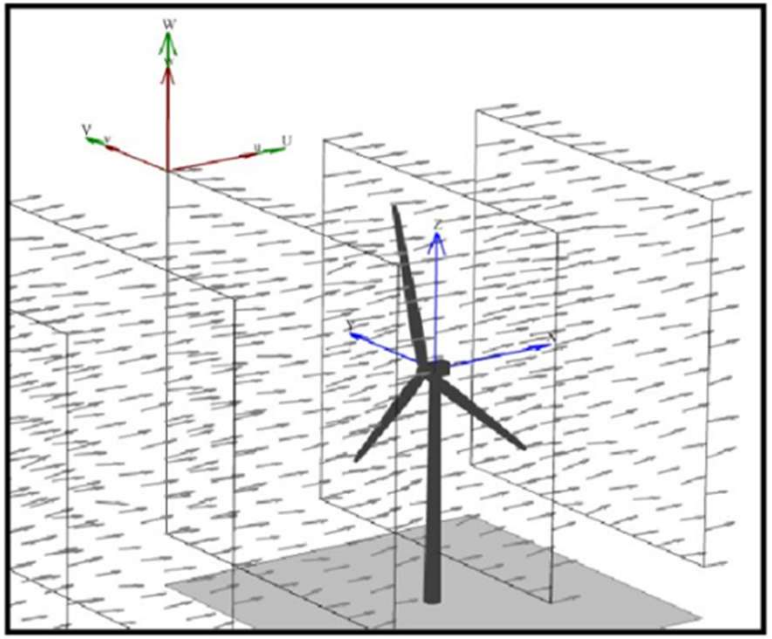
Thermal stratification (Monin–Obukhov) alters $\alpha(t)$. Relevant for large rotors where top/bottom speed ratio $> 20\%$.

Wake effects

Upstream turbine wakes: added turbulence and velocity deficit — modelled as modified inlet statistics.

WIND VELOCITY AS A STATIONARY GAUSSIAN PROCESS

PhD Simon QUERNE (Nov. 2024 – Nov. 2027)
Supervision: Laurent DUMAS (UVSQ), Charles Tillier (UVSQ), **Jean-François LECOMTE (IFPEN)**



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IEC 61400-1 Ed.3 Eq.(A.10); Kaimal et al. (1972)

UNCERTAIN INPUT PARAMETERS

PhD Simon QUERNE (Nov. 2024 – Nov. 2027)
Supervision: Laurent DUMAS (UVSQ), Charles Tillier (UVSQ), **Jean-François LECOMTE (IFPEN)**

The stationary model maps a parameter vector $\boldsymbol{\theta}$ to a wind field realisation. Non-stationarity is introduced by letting $\boldsymbol{\theta}$ vary in time or carry uncertainty:

$$\mathbf{U}(x, y, t\,;\, \boldsymbol{\theta}(t)), \qquad \boldsymbol{\theta}(t) = \left[U_{\text{hub}}(t),\ \sigma_u(t),\ L_u(t),\ \alpha(t),\ \varepsilon_{\text{EOG}}(t)\right]$$

$U_{\text{hub}}(t)$	$\sigma_u(t)$	$L_u(t)$	$\alpha(t)$	$\varepsilon_{\text{EOG}}(t)$
Time-varying mean speed. Weibull distribution or slow AR(1) drift.	Turbulence std dev ($I_u = \sigma_u/U_{\text{hub}}$). IEC class A/B/C or log-normal.	Integral length scale. Controls spectral peak. Uncertain from measurements.	Shear exponent (power law). Varies with atmospheric stability (Obukhov L).	Extreme-event indicator. Binary or Poisson arrival for EOG, ECD, etc.

Key remark

$\boldsymbol{\theta}(t)$ can be treated as a **random variable** (uncertainty analysis) or as a **stochastic process** itself (non-stationary model).

TOWARDS A LEARNED WIND FIELD GENERATOR (1)

PhD Simon QUERNE (Nov. 2024 – Nov. 2027)
Supervision: Laurent DUMAS (UVSQ), Charles Tillier (UVSQ), **Jean-François LECOMTE (IFPEN)**

Non-stationary **TurbSim** pipeline relies on hand crafted physical priors (θ) and remains costly for large MC ensembles.
A **data-driven generator** trained on realistic wind fields could replace or complement it.

Denoising Diffusion Probabilistic Model (DDPM)

A DDPM learns to reverse a Markov diffusion process:

$$p_\phi(\mathbf{U}^{(0)}) = \int p(\mathbf{U}^{(T)}) \prod_{t=1}^{T} p_\phi(\mathbf{U}^{(t-1)} \mid \mathbf{U}^{(t)})\, \mathrm{d}\mathbf{U}^{(1:T)}$$

The reverse step is parametrised by a neural network $\boldsymbol{\epsilon}_\phi(\mathbf{U}^{(t)}, t)$ trained to denoise a Gaussian-corrupted wind field.

Physical constraints

The generation is **conditioned** on $\boldsymbol{\theta}$ (mean speed, turbulence class, shear...) to enforce IEC-compatible statistics while capturing non-stationary structure beyond the Kaimal model.

DDPM WIND GENERATOR: PROPOSED WORKFLOW (1)

PhD Simon QUERNE (Nov. 2024 – Nov. 2027)
Supervision: Laurent DUMAS (UVSQ), Charles Tillier (UVSQ), **Jean-François LECOMTE (IFPEN)**

Training phase (offline, once)

1. Collect a dataset of wind fields $\{\mathbf{U}_i^{(0)}, \boldsymbol{\theta}_i\}$ from TurbSim runs or measurements.
2. Train the conditional DDPM $\boldsymbol{\epsilon}_\phi(\cdot, t, \boldsymbol{\theta})$ to minimise $\mathcal{L}(\phi)$.
3. Validate generated spectra against Kaimal PSD and spatial coherence targets.

Inference phase (online, per realisation)

1. Specify $\boldsymbol{\theta}$ (possibly non-stationary: $\boldsymbol{\theta}(t)$).
2. Sample $\mathbf{U}^{(T)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$.
3. Run T reverse denoising steps conditioned on $\boldsymbol{\theta}$.
4. Output: one realisation $\mathbf{U}(x, y, t; \boldsymbol{\theta})$.

NON-STATIONARY WIND FIELD GENERATION

Can we do the same or better with dedicated and taylored SDEs ?

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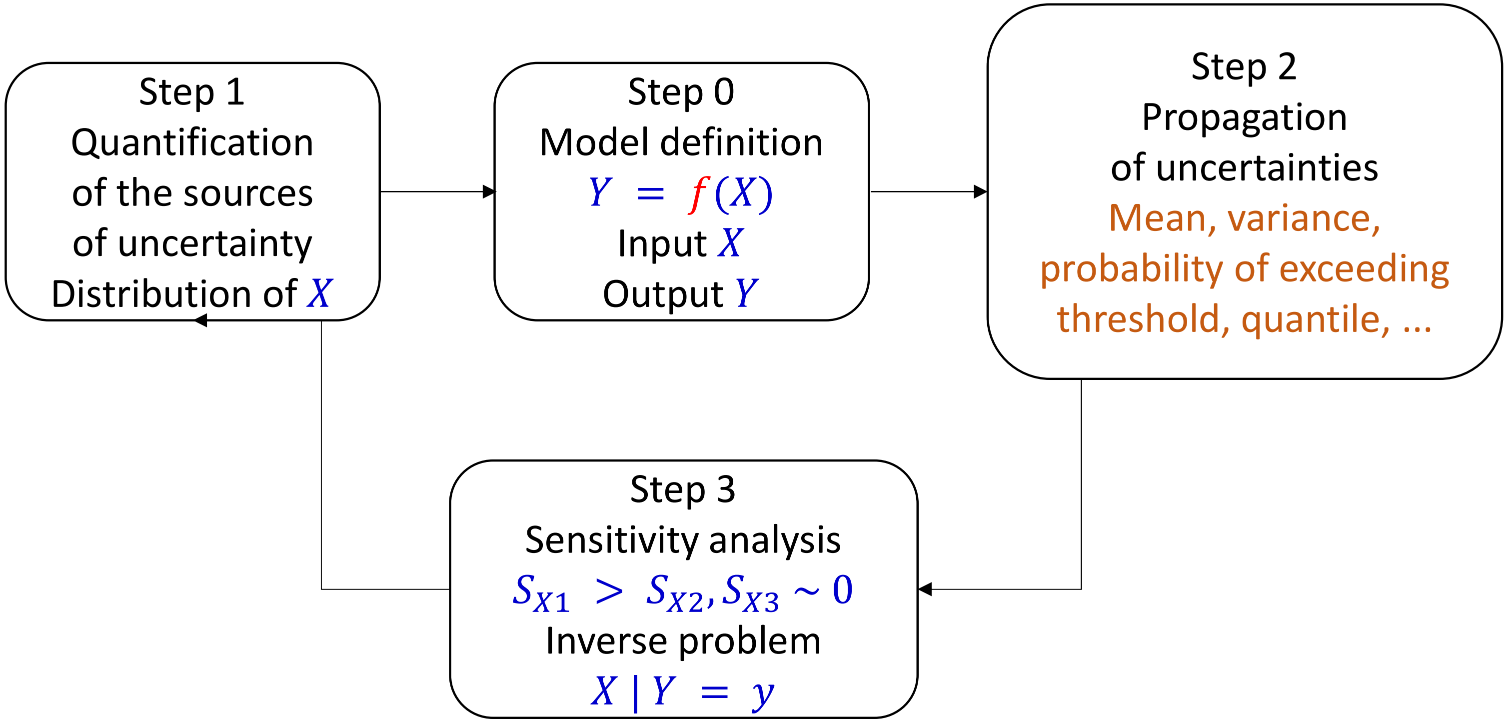
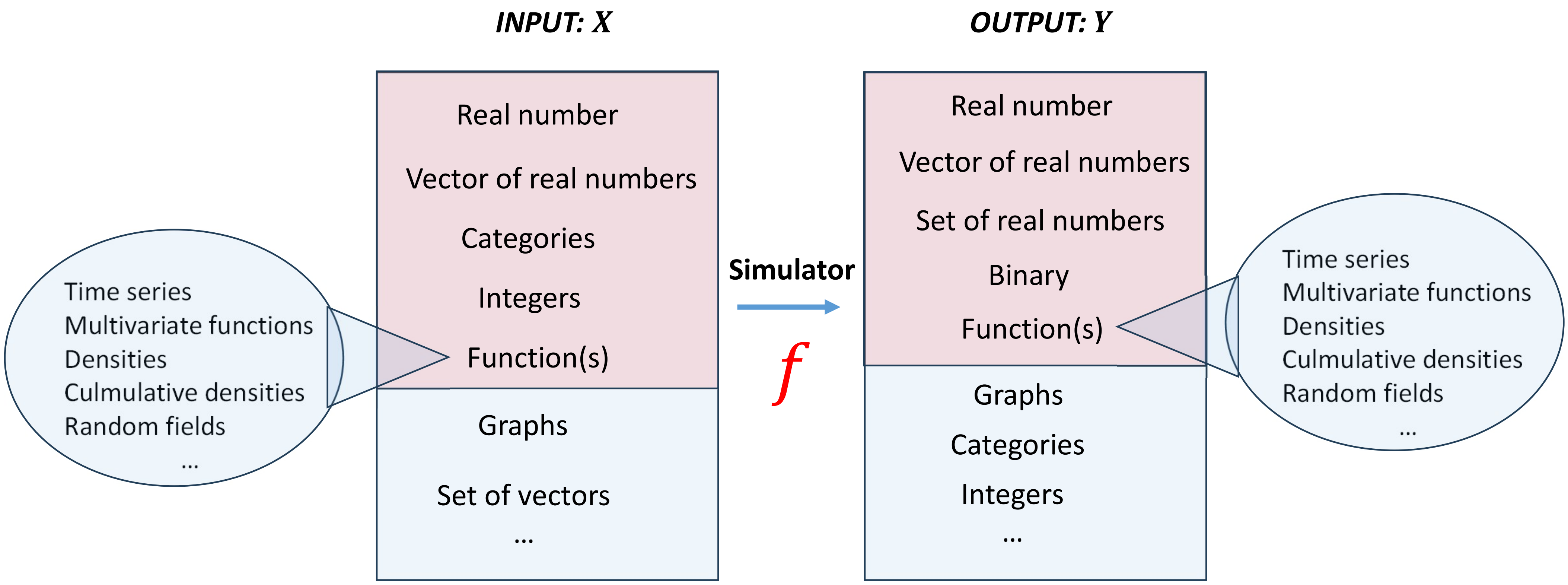
UNCERTAINTY QUANTIFICATION PIPELINE



Extract from a lecture, An introduction to Uncertainty Quantification, by Josselin Garnier.
RT-UQ workshop 12 and 13 November 2025 at Institut Henri Poincaré, Paris, France. https://uq.math.cnrs.fr/

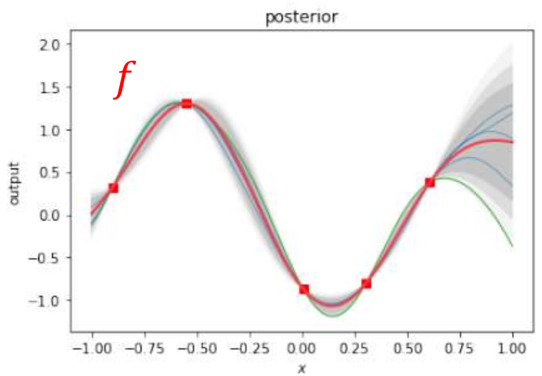
UNCERTAINTY QUANTIFICATION – INPUTS AND OUTPUTS



FRUGAL LEARNING

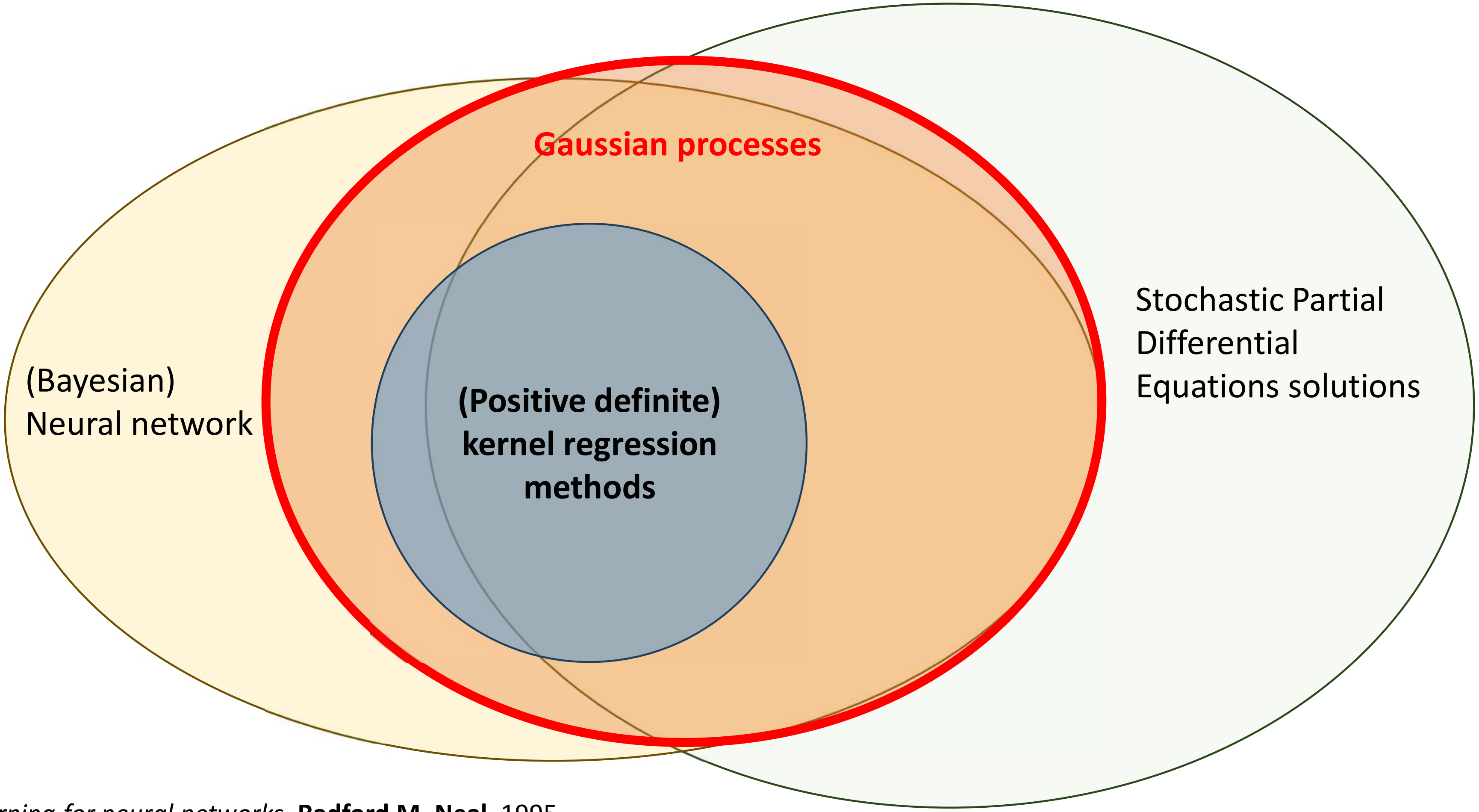
- **Complex numerical model** representing a physical phenomena with high fidelity
- Computational time constraint imposes an access to a **reduced number of simulation**
- **UQ analysis are costly in simulations**
- Introduction of a **surrogate model**: Gaussian processes → Probabilistic model ease uncertainty modelling and quantification

$f \overset{\sim}{\longleftrightarrow} F_n$ Gaussian process built on n data points , f is a realization of F_n



F_n

MACHINE LEARNING AND STOCHASTIC MODELLING - SKETCH



Bayesian learning for neural networks, **Radford M. Neal**, 1995.
Neural networks and quantum field theory, **J. Halverson, A. Maiti and K. Stoner**, 2020

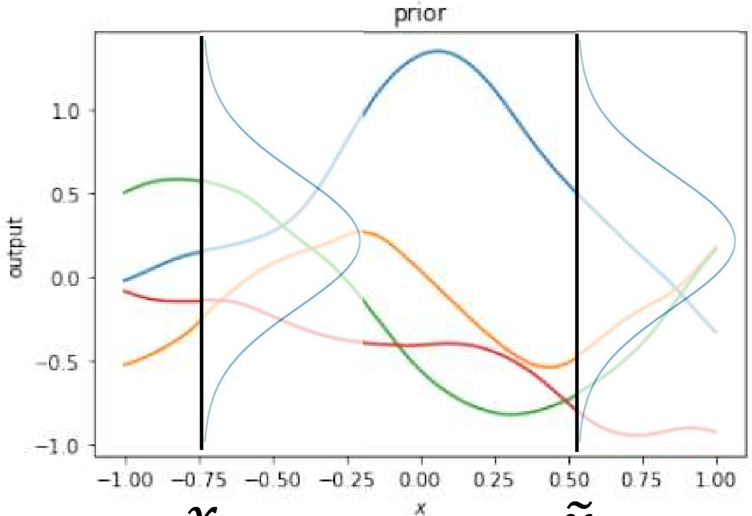
GAUSSIAN PROCESS MODEL FROM PRIOR TO POSTERIOR

Design of experiments

$$\mathcal{F}_n = \{F(x_1) = f(x_1), \ldots, F(x_n) = f(x_n)\}$$

Prior model

$$F \sim N(\mu, k)$$



x $\tilde{x}$

$$Cov\big(F(x), F(\tilde{x})\big) = k(x - \tilde{x})$$

Posterior model

$$F_n = F|\mathcal{F}_n \sim N(\mu_n, k_n)$$

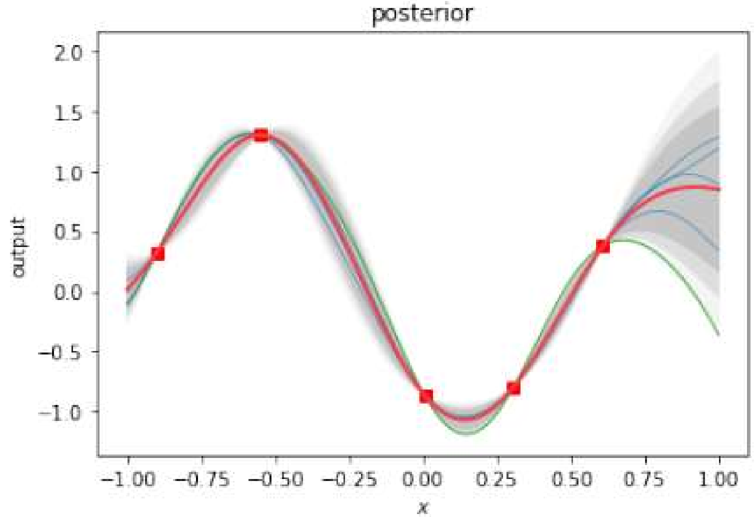


Figure: TwoBumps$(x) = -(0.7x + \sin(5x+1) + 0.1\sin(10x))$.

$A|B \Leftrightarrow A$ knowing B

QUANTITIES OF INTEREST

$$Y = f(X) \qquad \begin{aligned}(\Omega, \mathbb{P}) &\to (\mathbb{H}, \mathbb{P}_X)\\ \omega &\to Y(\omega)\end{aligned}$$

Interesting question, fully informative
What is the distribution of Y ?

But often we are interested in a specific quantity of interest function of Y

$$\mathcal{O}(Y) \to \mathcal{O}_f$$

"If you possess a restricted amount of information for solving some problem, try to solve the problem directly and never solve a more general problem as an intermediate step.
It is possible that the available information is sufficient for a direct solution but is insufficient for solving a more general intermediate problem."

V.N. Vapnik, Statistical Learning Theory, p-12, Wiley, 1998

QUANTITIES OF INTEREST

$$Y = f(X) \qquad \begin{array}{c}(\Omega, \mathbb{P}) \to (\mathbb{H}, \mathbb{P}_X) \\ \omega \to Y(\omega)\end{array}$$

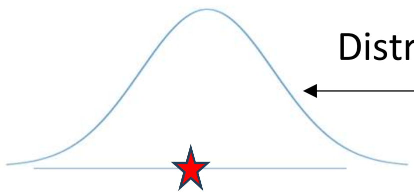
Interesting question, fully informative
What is the distribution of Y ?

But often we are interested in a specific quantity of interest function of Y

$$\mathcal{O}(Y) \to \mathcal{O}_f$$

Step 2
Propagation
of uncertainties
Mean, variance,
probability of exceeding
threshold, quantile,
Sensitivity analysis
Inverse problem
$\mathcal{O}_f \to \mathcal{O}_{F_n}$

$$\mathbb{E}_X\big(f(X)\big) = \mathcal{O}_f \to \mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X))$$



Distribution of $\mathcal{O}_{F_n}$

$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X))$
Random variable

$$\mathcal{O}_f = \mathbb{E}_X(f(X))$$

SEQUENTIAL UNCERTAINTY REDUCTION PIPELINE

My goal: $\mathcal{O}_f$

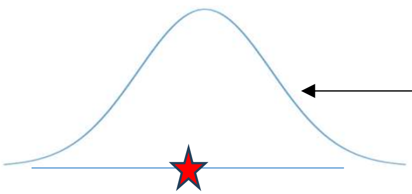
Current DoE: $\mathcal{X}_n$ used to build the stochastic model $\mathcal{O}_{F_n}$

Define (global) uncertainty measure $\mathcal{H}_n$

Go from $\mathcal{X}_n$ to $\mathcal{X}_n \cup \{x\}$ such that the expected uncertainty $\mathcal{H}_{n+1}$ is minimal i.e.

$$argmin_x \, \mathcal{H}_{n+1}(x)$$

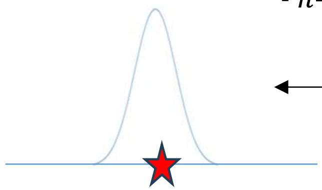
Distribution of $\mathcal{O}_{F_n}$



$\mathcal{O}_f = \mathbb{E}_X(f(X))$

$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X))$
Random variable

Distribution of $\mathcal{O}_{F_{n+1}}$



$\mathcal{O}_f = \mathbb{E}_X(f(X))$

$\mathcal{O}_{F_{n+1}} = \mathbb{E}_X(F_{n+1}(X))$
Random variable

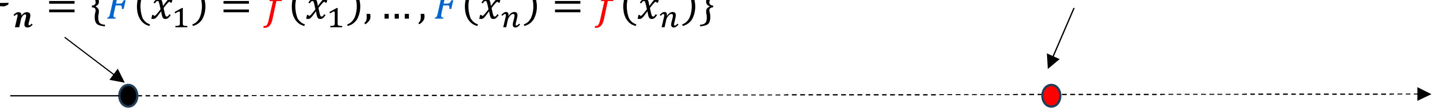
STEPWISE UNCERTAINTY APPROACH: **SUR**

$\mathcal{X}_n = \{x_1, \dots, x_n\}$

$\boldsymbol{\mathcal{F}_n} = \{F(x_1) = f(x_1), \dots, F(x_n) = f(x_n)\}$

$\mathcal{X}_{n+1} = \{x_1, \dots, x_n, x\}$

$\boldsymbol{\mathcal{F}_{n+1}} = \boldsymbol{\mathcal{F}_n} \cup \{F(x)\}$



H_n

Global Uncertainty measure

$H_{n+1}(x)$

$F_n(x) = F(x) | \boldsymbol{\mathcal{F}_n} \sim N(\mu_n(x), \sigma_n^2(x))$

Reducing expected uncertainty

$$\underset{x}{argmin}\, \mathbb{E}(H_{n+1}(x))$$

Advantages of SUR strategies

- Present « smoother » criteria to optimize
- Show **better performances than "classical"** strategies (Bect [2012])

Obstacles linked to SUR strategies

- If not analytical, criteria can be expensive to estimate requiring GP conditional simulations. Few solutions:
  - quadrature methods
  - simplifying approximation

QUANTITIES OF INTEREST

$Y = f(X)$ $\qquad (\Omega, \mathbb{P}) \to (\mathbb{H}, \mathbb{P}_X)$

$\omega \to Y(\omega)$

Interesting question, fully informative
What is the distribution of Y ?

But often we are interested in a specific quantity of interest function of Y

$$\mathcal{O}(Y) \to \mathcal{O}_f$$

<u>Standard strategies</u>

Add sequentially points to the learning design:

- for improving F_n as a model of f
- For improving a goal oriented heuristic criteria
- **For reducing the « real » uncertainty on the seeked quantity**

Set estimation
Output: sets

$\{x, f(x) < s\}$

$\forall i, \{x, f_i(x) < s_i\}$

C. Duhamel PhD

$\{u, \mathbb{E}_X(f(u, X)) < s\}$

R. El Amri PhD

$\{x, \mathbb{I}(x) = NaN\}$

M. Menz PostDoc

Chance constraint optimization
Output: number, function

$\min_u C(u)$

s.t $c(u) = \mathbb{P}_X(g \circ f(u, X) > 0) > 1 - \epsilon$

A. Cousin PhD $\qquad \mathbb{E}_X(g \circ f(X))$

Moment, probability estimation
Output: number

$\mathbb{P}_X(g \circ f(X) > 0)$

A. Murangira, V. Breaz PostDoc

Bayesian inversion, calibration

Output: distribution, Random Var

$X|Y_{real,m}Y_m$ or $p(X|Y_{real,m}, Y_m)$

$C_f(\mathcal{X}_m) = p_f(X|\mathcal{X}_m)$ or $X|\mathcal{X}_m$

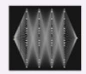
A. Hirvoas, A. Barry PhD

Sensitivity analysis
Output: vector

$S^f(X)$

E. Bartok PhD

Metamodelling
Output: function



E. Rondeaux PhD

Space-filling design
Output: finite set of functions

$\mathcal{X}_n$ L. Calzolari PhD

QUANTITIES OF INTEREST

$Y = f(X)$ $\qquad$ $(\Omega, \mathbb{P}) \to (\mathbb{H}, \mathbb{P}_X)$
$\omega \to Y(\omega)$

Interesting question, fully informative
What is the distribution of Y ?

But often we are interested in a specific quantity of interest function of Y

$$\mathcal{O}(Y) \to \mathcal{O}_f$$

<u>Standard strategies</u>

Add sequentially points to the learning design:

- for improving F_n as a model of f
- For improving a goal oriented heuristic criteria
- **For reducing the « real » uncertainty on the seeked quantity**

Set estimation
Output: sets

$\{x, F_n(x) < s\}$

$\forall i, \{x, F_{n,i}(x) < s_i\}$

C. Duhamel PhD

$\{u, \mathbb{E}_X(F_n(u, X)) < s\}$

R. El Amri PhD

$\{x, \mathbb{I}_n^f(x) = NaN\}$

M. Menz PostDoc

Bayesian inversion, calibration

Output: distribution, Random Var

$X|Y_{real,m}Y_m$ or $p(X|Y_{real,m}, Y_m)$

$C_f(\mathcal{X}_m) = p_f(X|\mathcal{X}_m)$ or $X|\mathcal{X}_m$

A. Hirvoas, A. Barry PhD

Chance constraint optimization
Output: number, function

$\min_u C(u)$
s.t $c(u) = \mathbb{P}_X(g \circ F_n(u, X) > 0) > 1 - \epsilon$

A. Cousin PhD $\qquad$ $\mathbb{E}_X(g \circ F_n(X))$

Moment, probability estimation
Output: number

$\mathbb{P}_X(g \circ F_n(X) > 0)$

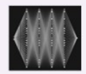
A. Murangira, V. Breaz PostDoc

Sensitivity analysis
Output: vector

$S^{F_n}(X)$

E. Bartok PhD

Metamodelling
Output: function



E. Rondeaux PhD

Space-filling design
Output: finite set of functions

$\mathcal{X}_n$ L. Calzolari PhD

QUANTITIES OF INTEREST

$Y = f(X)$ $\quad (\Omega, \mathbb{P}) \to (\mathbb{H}, \mathbb{P}_X)$, $\quad \omega \to Y(\omega)$

Interesting question, fully informative
What is the distribution of Y ?

But often we are interested in a specific quantity of interest function of Y

$$\mathcal{O}(Y) \to \mathcal{O}_f$$

Standard strategies

Add sequentially points to the learning design:

- for improving F_n as a model of f
- For improving a goal oriented heuristic criteria
- **For reducing the « real » uncertainty on the seeked quantity**

2.

Set estimation
Output: sets

$\{x, F_n(x) < s\}$

$\forall i, \{x, F_{n,i}(x) < s_i\}$

C. Duhamel PhD

$\{u, \mathbb{E}_X(F_n(u, X)) < s\}$

R. El Amri PhD

$\{x, \mathbb{I}_n^f(x) = NaN\}$

M. Menz PostDoc

1.

Expectation estimation for Chance constraint optimization

$$\min_u C(u)$$
$$\text{s.t } c(u) = \mathbb{P}_X(g \circ F_n(u, X) > 0) > 1 - \epsilon$$

A. Cousin PhD $\quad \mathbb{E}_X(g \circ F_n(X))$

Moment, probability estimation
Output: number

$\mathbb{P}_X(g \circ F_n(X) > 0)$

A. Murangira, V. Breaz PostDoc

Bayesian inversion, calibration

Output: distribution, Random Var

$X|Y_{real,m} Y_m$ or $p(X|Y_{real,m}, Y_m)$

$C_f(\mathcal{X}_m) = p_f(X|\mathcal{X}_m)$ or $X|\mathcal{X}_m$

A. Hirvoas, A. Barry PhD

Sensitivity analysis
Output: vector

$S^{F_n}(X)$

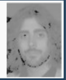
E. Bartok PhD

Metamodelling
Output: function

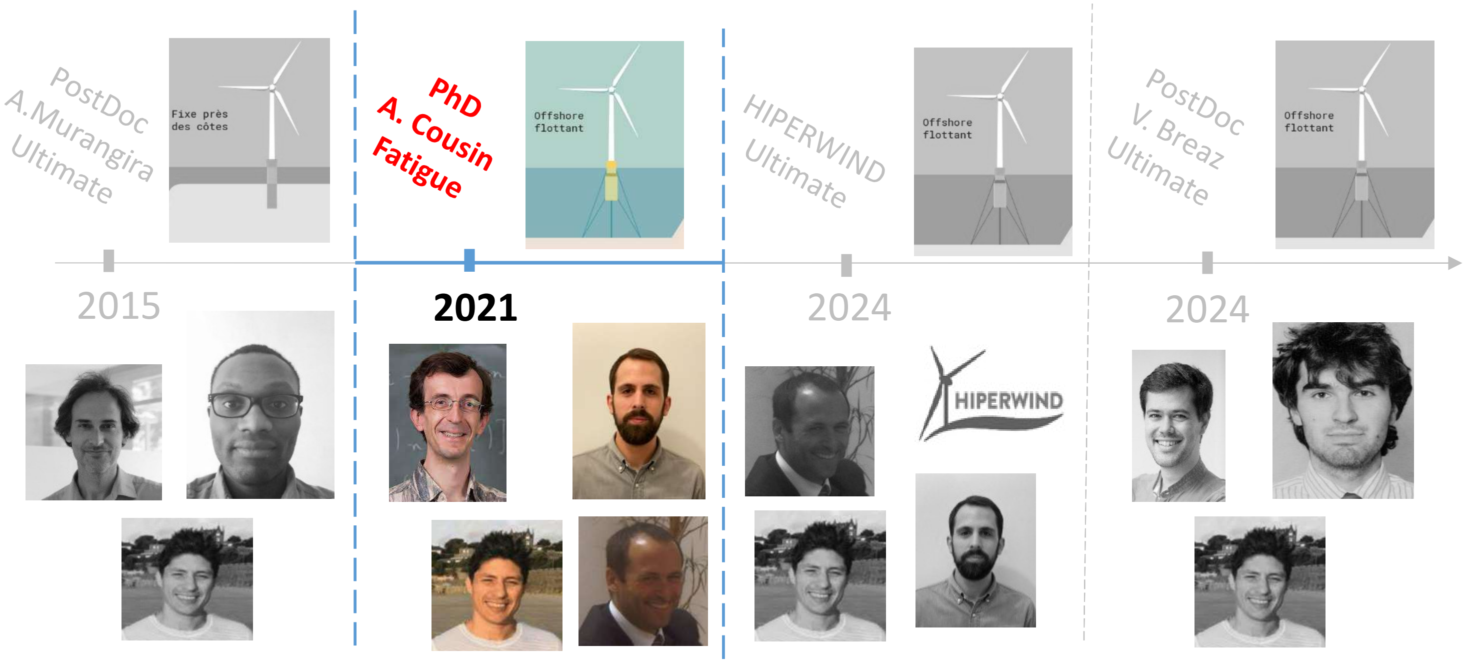
E. Rondeaux PhD

Space-filling design
Output: finite set of functions

$\mathcal{X}_n$ L. Calzolari PhD



ROBUST DESIGN AND RELIABILITY ANALYSIS



IFPEN | CMAP - IFPEN | IFPEN | INRIA - IFPEN

Chance constraint optimization

Output : number, function

$$\min_{u} C(u)$$
$$\text{s.t } c(u) = \mathbb{P}_X(g \circ f(u, X) > 0) > 1 - \epsilon$$

A. Cousin PhD $\mathbb{E}_X(g \circ f(X))$

Moment, probability estimation
Output : number

$\mathbb{P}_X(g \circ f(X) > 0)$
A. Murangira, V. Breaz PostDoc

CHANCE CONSTRAINT OPTIMIZATION - A. COUSIN PHD

Optimize the cost of morring system under fatigue and feasability constraints

$$\min_{d\in\Omega_d} cost(d) \quad \text{such that}$$

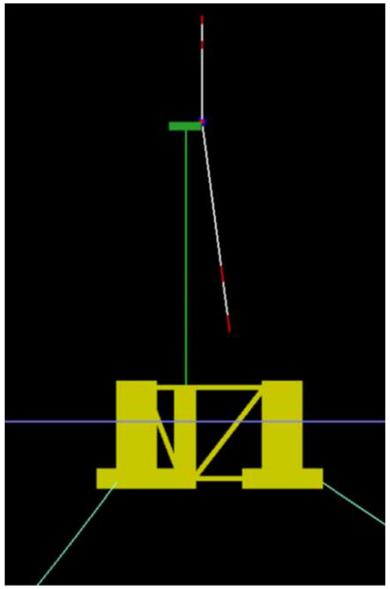
$$\mathbb{P}_{X_p,\eta}\left(\max_{t\in[0,T]} |\mathcal{S}(d,X_p;t)| > \mathcal{S}_{\max}\right) < 10^{-4}$$

$$\mathbb{P}_{X_p,\eta}\left(\min_{t\in[0,T]} \mathcal{T}^l(d,X_p;t) < 0\right) < 10^{-4} \quad , l = 1,2,3$$

$$\mathbb{P}_{X_{d_2},X_p,X_R,\eta}\left(\int_0^T \mathcal{D}^l(d,X_{d_2},X_p;t)\,dt > X_R\right) < 10^{-4} \quad , l = 1,2,3.$$

$\boldsymbol{d}$: length to the mooring, mass per unit, position of lines to columns
Uncertainties: Azimuth, damping and fatigue law coefficients

NREL 5MW turbine on DeepCWind floater



DEEPLINES^{TM}

The sea elevation **stationary Gaussian process** defined by its **spectral density** (JONSWAP)

CHANCE CONSTRAINT OPTIMIZATION - A. COUSIN PHD

Optimize the cost of morring system under fatigue and feasability constraints

$$\min_{d \in \Omega_d} cost(d) \quad \text{such that}$$

$$\mathbb{P}_{X_p,\eta}\left(\max_{t\in[0,T]} |\mathcal{S}(d, X_p; t)| > \mathcal{S}_{\max}\right) < 10^{-4}$$

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$$\mathbb{P}_{X_{d_2},X_p,X_R,\eta}\left(\int_0^T \mathcal{D}^l(d, X_{d_2}, X_p; t)\, dt > X_R\right) < 10^{-4} \quad , l = 1,2,3.$$

Extreme value theory
(linearization of the equation of motion)

$$\min_{d \in \Omega_d} cost(d) \quad \text{such that}$$

$$\mathbb{E}_{X_p}\left[F_\epsilon\left(\sum_{j=1}^{7} \exp\left(a_{Tp^j}(d, X_p, s^j)^2 - \frac{a_{Tp^j}(d, X_p, s^j)\left(\mathcal{S}_{\max} + \mu_{\mathcal{S}}(d, X_p, s^j)\right)}{\sqrt{m_{\mathcal{S},0}(d, X_p, s^j)}}\right)\right)\right] < 10^{-4}$$

$$\mathbb{E}_{X_p}\left[F_\epsilon\left(\sum_{j=1}^{7} \exp\left(b^l_{Tp^j}(d, X_p, s^j)^2 - \frac{b^l_{Tp^j}(d, X_p, s^j)\mu_{\mathcal{T}^l}(d, X_p, s^j)}{\sqrt{m_{\mathcal{T}^l,0}(d, X_p, s^j)}}\right)\right)\right] < 10^{-4} \; , \; l = 1,2,3$$

$$\mathbb{E}_{X_p}\left[\Phi\left(\frac{\log\left(T \sum_{j=1}^{7} p^j E_{\mathcal{T}^l}(d, X_p, s^j)\right) - (\mu_R + \mu_{d_2})}{\sqrt{\sigma_R^2 + \sigma_{d_2}^2}}\right)\right] < 10^{-4} \; , \; l = 1,2,3. \quad (5.6)$$

- New quantity of interest:

$$\mathbb{E}_X(g \circ F_n(X))$$

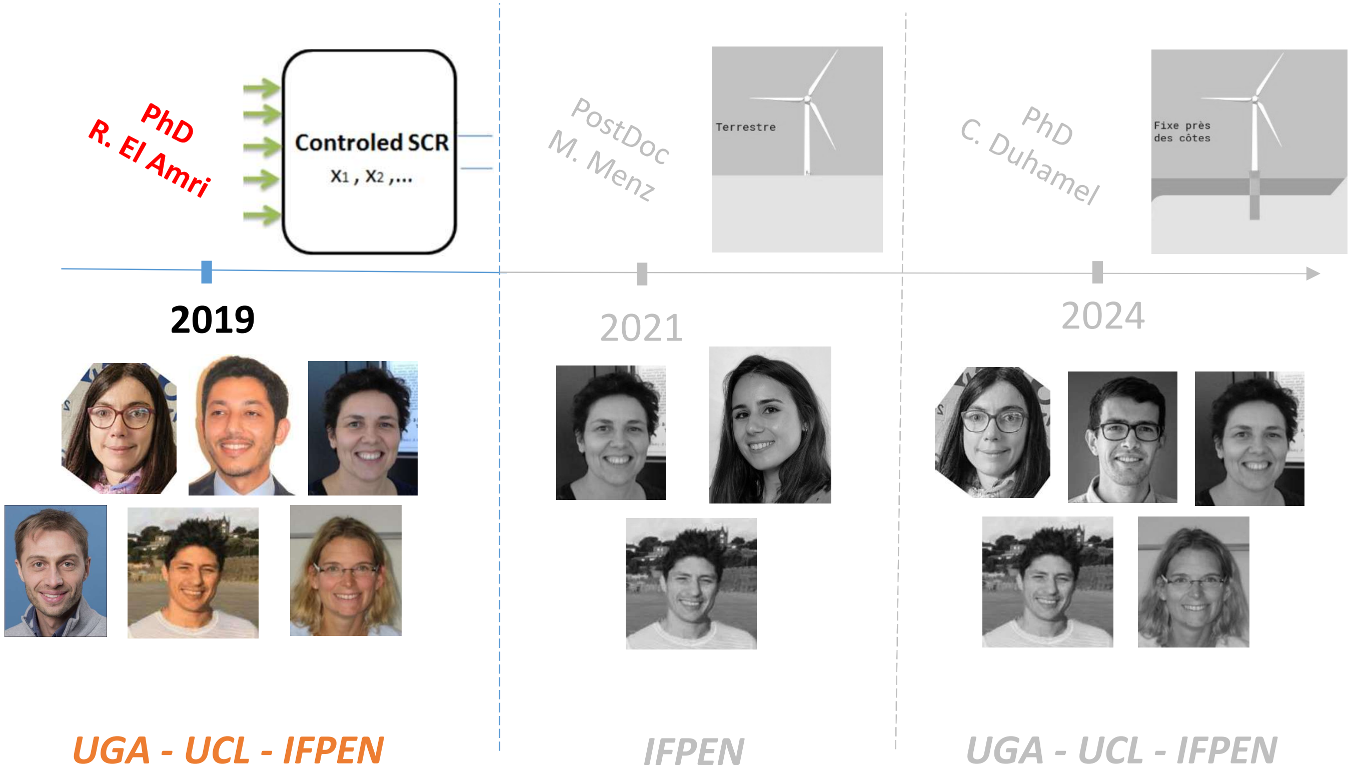
- Improving a goal oriented heuristic criteria:

$$\underset{x}{argmax}\, p_X(x)\left[g\left(\mu_n^+(x)\right) - g\left(\mu_n^-(x)\right)\right]$$

with $\mu_n^{\pm} = \mu_n \pm 2\sigma_n$

$$\underset{\substack{u \in \mathcal{U} \\ v \in \{v_i, i=1,\dots,K\}}}{\text{argmax}} \; p_{\mathcal{U}}(u)\left[g_1\left(\mu^+(u,v) + \sum_{v_i \neq v} \mu(u, v_i)\right) - g_1\left(\mu^-(u,v) + \sum_{v_i \neq v} \mu(u, v_i)\right)\right]$$

SET ESTIMATION



Set estimation
Output: sets

$\{x, f(x) < s\}$

$\forall i, \{x, f_i(x) < s_i\}$

C. Duhamel PhD

$\{\mathbf{u}, \mathbb{E}_X(f(\mathbf{u}, X)) < s\}$

R. El Amri PhD

$\{x, \mathbb{I}^f(x) = NaN\}$

M. Menz PostDoc

ROBUST SET ESTIMATION

$$\mathcal{O}_{\mathcal{L}\circ F_n} = \{\mathrm{u}, \mathbb{E}_X\big(F_n(u,X)\big) < s\} = \{\mathrm{u}, [\mathcal{L}\circ F_n](u) < s\}$$

$$[\mathcal{L}\circ F_n](u) = \mathbb{E}_X\big(F_n(u,X)\big)$$

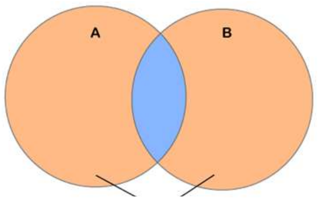
Is a known Gaussian process $\mathcal{GP}(\mu_n^{\mathcal{L}}, \sigma_n^{\mathcal{L}})$

- Uncertainty measure: Vorob'ev deviation (*[Molchanov, 2006]*)

$$\pi_n(u) = \mathbb{P}([\mathcal{L}\circ F_n](u) < s)$$

$$d_V(\mathcal{O}_{[\mathcal{L}\circ F_n]}) = \mathbb{E}_{[\mathcal{L}\circ F_n]}\Big(\lambda\big(\mathcal{O}_{[\mathcal{L}\circ F_n]}\Delta \mathcal{Q}_{n,\alpha_n^*}\big)\Big)$$

$$\mathbb{E}_{F_n}\Big(\lambda\big(\mathcal{O}_{F_n}\Delta \mathcal{Q}_{n,\alpha_n^*}\big)\Big) = \int_{\mathcal{Q}^c_{n,\alpha_n^*}} \pi_n(u)du + \int_{\mathcal{Q}_{n,\alpha_n^*}} 1-\pi_n(u)du$$



$A\Delta B$

- SUR criteria on Vorob'ev deviation:

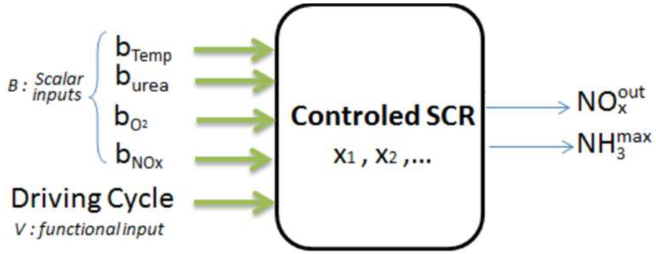
$$\underset{u}{argmin}\ \mathbb{E}_{[\mathcal{L}\circ F](u)|\mathcal{F}_n}\Big[d_V\big(\mathcal{O}_{[\mathcal{L}\circ F]_{n+1}}\big)\Big]$$

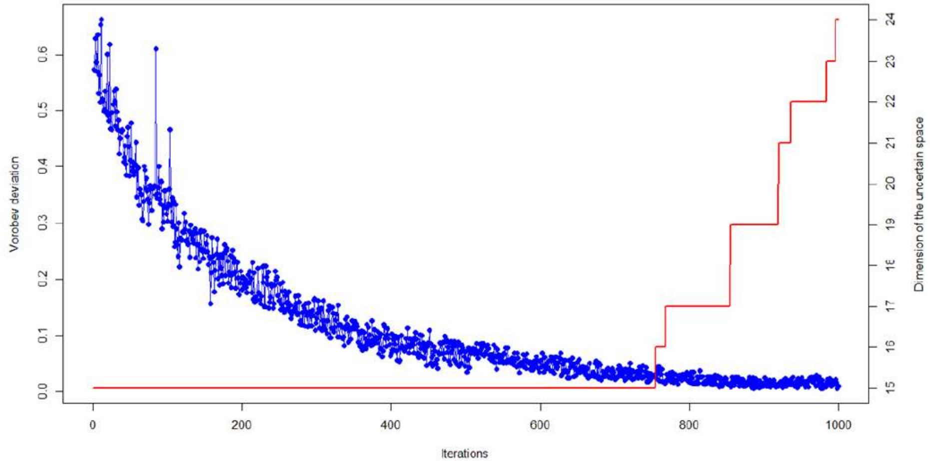
$$\underset{x}{argmin}\ Var\big[[\mathcal{L}\circ F]_{n+1}(u_{n+1})\big]$$

$$\mathcal{Q}_{n,\alpha_n^*} = \mathbb{P}(\pi_n(u) > \alpha_n^*)$$
$$= \{u,\ \mu_n^{\mathcal{L}}(u) + \Phi^{-1}(\alpha_n^*)\sigma_n^{\mathcal{L}}(u) \le s\}$$

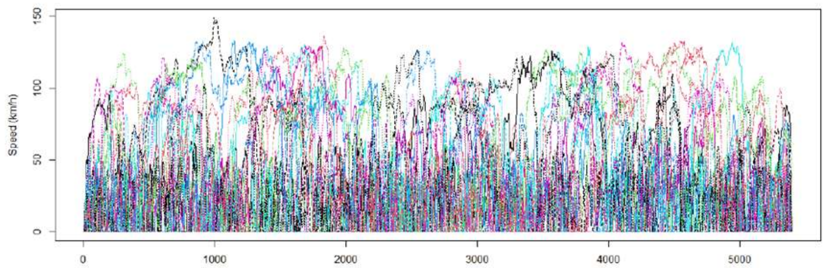
α_n^* such that $\lambda\big(\mathcal{Q}_{n,\alpha_n^*}\big) = \mathbb{E}_{F_n}\Big(\lambda\big(\mathcal{O}_{\mathcal{L}\circ F_n}\big)\Big)$

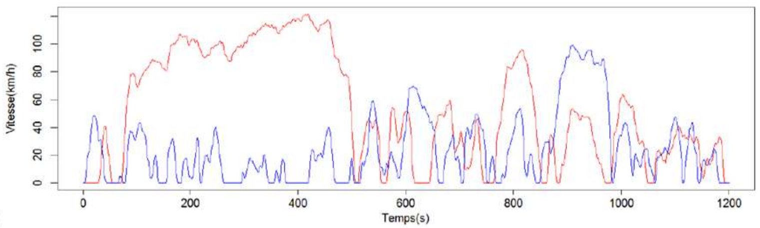
ROBUST SET ESTIMATION FOR SYSTEMS CONTROLE

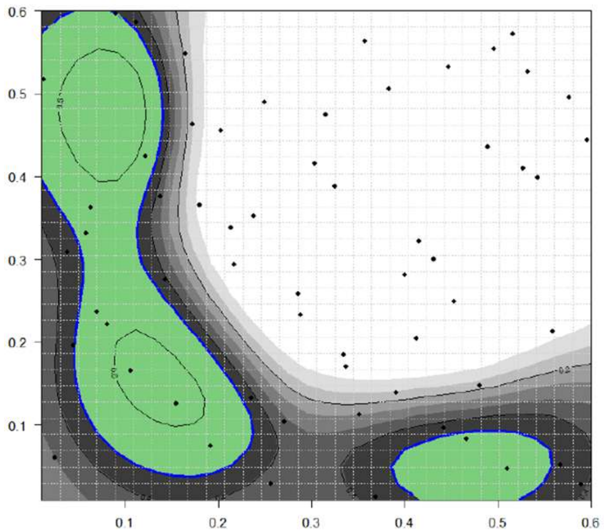


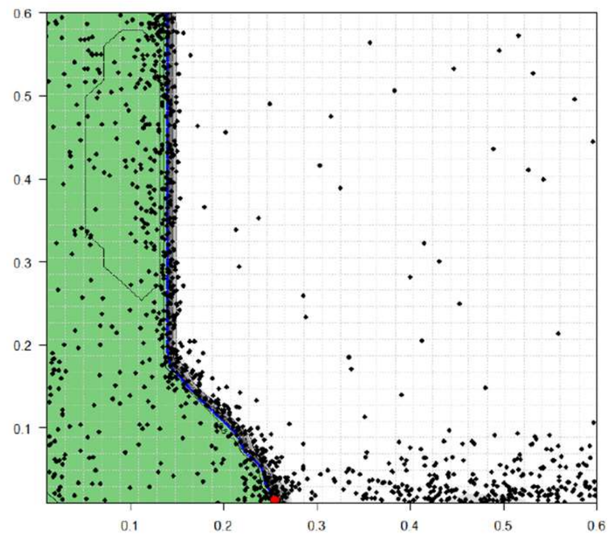


$$f : \begin{cases} \mathbb{X} \times \mathcal{V} & \to \mathbb{R} \\ (\mathbf{x}, \mathbf{V}) & \mapsto f(\mathbf{x}, \mathbf{V}) = \max_{t \in I} \mathrm{NH}_3(t) \end{cases}$$









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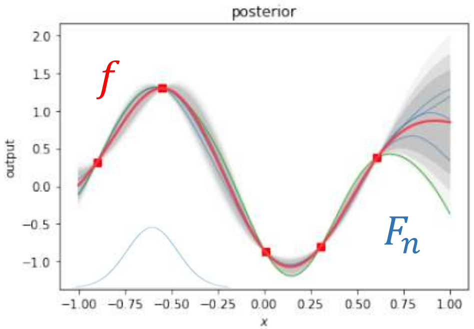
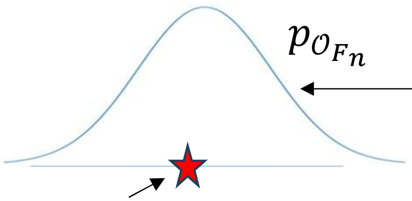
GOAL ORIENTED UNCERTAINTY REDUCTION



$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to \mathbb{R}$



$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X))$
Random variable

$\mathcal{O}_f = \mathbb{E}_X(f(X))$

reduction of the variance: $Var(\mathcal{O}_{F_n})$

stepwise uncertainty reduction (**SUR**) approach:

$$x_{n+1} = \underset{x}{argmin}\, \mathbb{E}_{Y(x)|\mathbf{Y}_n}\left[Var(\mathcal{O}_{F_{n+1}})\right]$$

$\mathcal{O}: L_2(\Omega; L_2(D)) \to L_2(\Omega, \mathbb{R})$

What about

$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X)^k)$

$\mathcal{O}_{F_n} = \mathbb{E}_X(g \circ F_n(X))$

$\mathcal{O}_{F_n} = \mathbb{P}_X(g \circ F_n(X) > 0)$

$\mathcal{O}_{F_n} = q_{g \circ F_n(X)}(\alpha)$

$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to L_2(\Omega, \mathbb{R}^d)$

What about

$\mathcal{O}_{F_n} = (S_1, S_2, S_{12}, \dots)$

$\mathcal{O}: L_2(\Omega; L_2(D)) \to (\Omega, \mathbb{S})$

What about

$\mathcal{O}_{F_n} = \{\mathrm{x}, g \circ F_n(x) < s\}$

$\mathcal{O}_{F_n} = \{\mathrm{u}, \mathbb{E}_X(g \circ F_n(u, X)) < s\}$

$\mathcal{O}_{F_n} = \{\mathrm{u}, \mathbb{P}_X(g \circ F_n(u, X) > 0) < \epsilon\}$

$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to L_2(\Omega, \mathbb{H})$

What about

$\mathcal{O}_{F_n} = G(\cdot, F_n)$

…

INTRODUCTION TO POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

$\mathcal{O}: L_2(\Omega; L_2(D)) \rightarrow L_2(\Omega; \mathbb{R})$, If $\mathcal{O}_{F_n} \in \mathbb{D}^{1,2}(\gamma_n)$ then

$$Var(\mathcal{O}_{F_n}) \leq \mathbb{E}_{F_n}\left(\left\|\mathcal{D}\mathcal{O}_{F_n}\right\|_{\mathbb{H}_n}^2\right)$$

- $\mathbb{H}_n$: Cameron-Martin space (RKHS associated to the covariance kernel) of F_n
- γ_n: posterior probability measure of the process, defined over an infinite-dimensional space of functional paths
- $\mathbb{D}^{1,2}(\gamma_n)$: completion of the set of smooth cylindrical random variables $\{\psi[F_n(x_1), \ldots, F_n(x_k)]\}$ w.r.t. the norm $\|S\|_{1,2} = \mathbb{E}_{\gamma_n}(S^2) + \mathbb{E}_{\gamma_n}\left(\|\mathcal{D}S\|_{\mathbb{H}_n}^2\right)$ named the « Sobolev-Watanabe » space.
- $\mathcal{D} : \mathbb{D}^{1,2}(\gamma_n) \rightarrow L^2(B, \gamma_n; \mathbb{H}_n)$ is the Malliavin derivative we apply to the random variable $\mathcal{O}_{F_n}$
- $\mathcal{D}\mathcal{O}_{F_n}$ is random variable with realization in $\mathbb{H}_n$
- $\mathcal{O}_{F_n}$ has to belong to the « Sobolev-Watanabe » space $\mathbb{D}^{1,2}(\gamma_n)$

- **We can generalize this type of results to the case of non-smooth operators** (for instance involving indicator functions) by using the notion of Watanabe distributions (in L. Schwartz sense) and isoperimetric inequalities.

David Nualart,The Malliavin calculus and related topics, Springer, 2006. Etc…

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to active learning of expectations

$$\mathcal{O}_{g\circ F_n} = \mathbb{E}_X(g \circ F_n(X))$$

First approach: reduce uncertainty on the **integrand** term $g \circ F_n(X)$

- **Heuristic criteria**

  $$\underset{x}{argmax}\, p_X(x)\left[g\left(\mu_n^+(x)\right) - g\left(\mu_n^-(x)\right)\right] \text{ with } \mu_n^\pm = \mu_n \pm 2\sigma_n$$

  Example:

  See *[thesis Alexis Cousin, 2021]*

- **Variance** based criteria

  $$\underset{x}{argmax}\, Var(g \circ F_n(x))$$

  Example:

  $$Var(\exp[F_n(x)]) = \exp\left(2\mu_n(x) + \sigma_n(x)\right)\left(\exp\left(\sigma_n(x)\right) + 1\right)$$

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to active learning of expectations

$$\mathcal{O}_{g\circ F_n} = \mathbb{E}_X(g \circ F_n(X))$$

Second approach: reduce uncertainty on the real objective $\mathcal{O}_{g\circ F_n}$

$$Var(\mathcal{O}_{g\circ F_n}) \leq \mathbb{E}_{F_n}\left(\left\|\mathcal{D}\mathcal{O}_{g\circ F_n}\right\|^2_{\mathbb{H}_n}\right)$$

Global uncertainty measure

SUR criteria:

$$\mathbb{E}_{F(x)|\mathcal{F}_n}\left(\mathbb{E}_{F_{n+1}}\left(\left\|\mathcal{D}\mathcal{O}_{g\circ F_{n+1}}\right\|^2_{\mathbb{H}_{n+1}}\right)\right) = \sigma_n(x)^{-2}\mathbb{E}_{X,X'}[k_n(X,x)k_n(X',x)A_{n+1}(X,X')]$$

$$A_{n+1}(X,X') = \mathbb{E}_{F(x)|\mathcal{F}_n}\left(\mathbb{E}_{F_{n+1}}[g'(F_{n+1}(X)\,)g'(F_{n+1}(X')\,)]\right)$$

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to active learning of expectations

For example: $g = Id$

$$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X))$$

Second approach: reduce uncertainty on the real objective $\mathcal{O}_{F_n}$

$$Var(\mathcal{O}_{F_n}) = \mathbb{E}_{F_n}\left(\left\|\mathcal{D}\mathcal{O}_{F_n}\right\|_{\mathbb{H}_n}^2\right)$$

- SUR criteria:

$$\mathbb{E}_{F(x)|\mathcal{F}_n}\left[Var(\mathcal{O}_{F_{n+1}})\right] = Var(\mathcal{O}_{F_{n+1}}) = \mathbb{E}_{F_{n+1}}\left(\left\|\mathcal{D}\mathcal{O}_{F_{n+1}}\right\|_{\mathbb{H}_{n+1}}^2\right) = \sigma_n(x)^{-2}\mathbb{E}_X\left[k_n(X,x)\right]^2$$

[Cousin et al, 2024]

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to active learning of expectations

For example: $g = \exp$

$$\mathcal{O}_{\exp\circ F_n} = \mathbb{E}_X(\exp \circ F_n(X))$$

Second approach: reduce uncertainty on the real objective $\mathcal{O}_{\exp\circ F_n}$

$$Var(\mathcal{O}_{\exp\circ F_n}) \leq \mathbb{E}_{F_n}\left(\left\|\mathcal{D}\mathcal{O}_{\exp\circ F_n}\right\|^2_{\mathbb{H}_n}\right)$$

- SUR criteria - one point:

$$\underset{x}{argmin}\, \mathbb{E}_{F(x)|\mathcal{F}_n}\left[\mathbb{E}_{F_{n+1}}\left(\left\|\mathcal{D}\mathcal{O}_{\exp\circ F_{n+1}}\right\|^2_{\mathbb{H}_{n+1}}\right)\right] = \underset{x}{argmax}\, \sigma_n(x)^{-2}\mathbb{E}_{X,X'}[k_n(X,x)k_n(X',x)A_n(X,X')]$$

$$A_n(X,X') = \exp\left[\mu_n(X) + \mu_n(X') + \frac{1}{2}(\sigma_n(X) + \sigma_n(X') + 2k_n(X,X'))\right]$$

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to active learning of expectations

- Non-stationarity issues

$$\mathcal{O}_f = \mathbb{E}_X(f(X)) \qquad \longrightarrow \qquad \mathcal{O}_{F_n} = \mathbb{E}_X(exp \circ F_n(X))$$

$$= \mathbb{E}_X\left(exp \circ \underbrace{log[f}_{\tilde{f}}(X)]\right)$$

$\tilde{f}$ → F_n

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

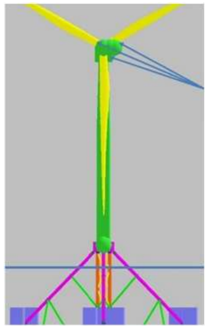
Application to active learning of expectations

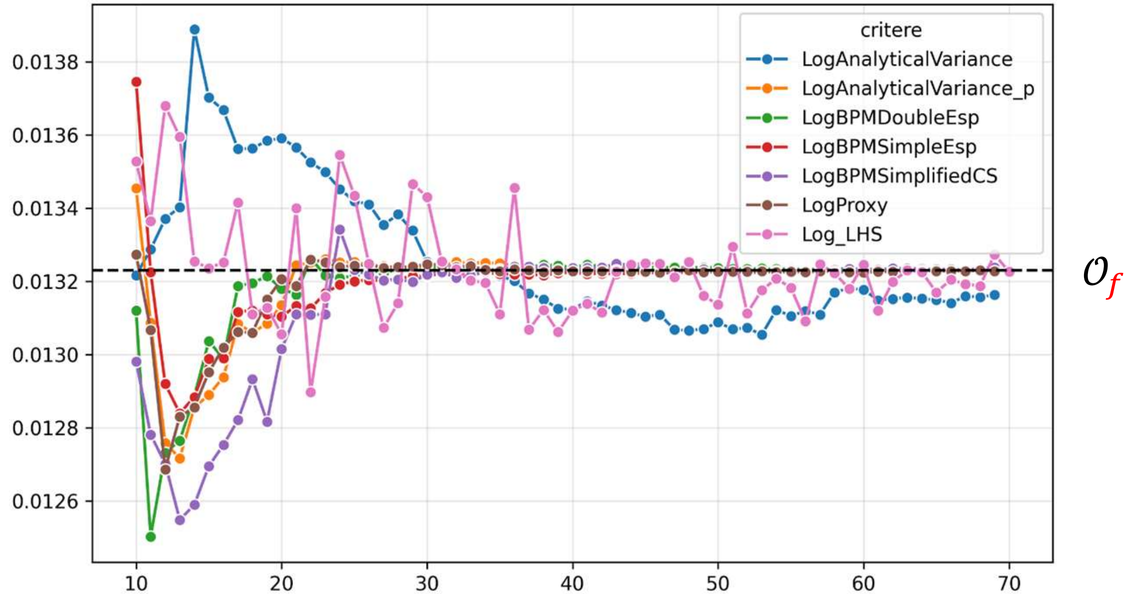
$$\mathcal{O}_{F_n} = \mathbb{E}_X(exp \circ F_n(X))$$

$\underset{x}{argmax}\, p_X(x)[g(\mu_n^+(x)) - g(\mu_n^-(x))]$	Proxy
$\underset{x}{argmax}\, p_X(x)^2 \exp(2\mu_n(x) + \sigma_n(x))(\exp(\sigma_n(x)) + 1)$	Analytical variance
$\underset{x}{argmax}\, \sigma_n(x)^{-1}\mathbb{E}_X\left[k_n(X,x)\exp\left(\mu_n(X) + \frac{1}{2}\sigma_n^2(X)\right)\right]$	Simple Exp
$\underset{x}{argmax}\, \sigma_n(x)^{-2}\mathbb{E}_X[k_n(X,x)^2]$	Simplified

- Estimation mean annual damage at the top of a mooring line.
- 6MW tension leg platform (TLP) wind turbine model.

- **2D input case:** $(\boldsymbol{U}, \boldsymbol{\sigma_U})$
- Initialization: 20 points
- Monte Carlo: 50000 points
- Iteration: 30, one point at the time





Double Exp. $\underset{x}{argmax}\, \sigma_n(x)^{-2}\mathbb{E}_{X,X'}\left[k_n(X,x)k_n(X',x)\exp\left[\mu_n(X) + \mu_n(X') + \frac{1}{2}(\sigma_n(X) + \sigma_n(X') + 2k_n(X,X'))\right]\right]$

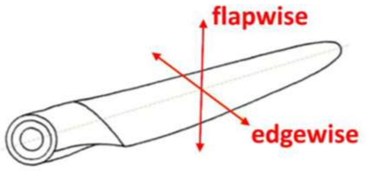
POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to active learning of expectations

$$\mathcal{O}_{F_n} = \mathbb{E}_X(exp \circ F_n(X))$$

- Estimation mean annual **flapwise** and **edgewise** blade root damage
- 6MW tension leg platform (TLP) wind turbine model.

- **6D case** $(\boldsymbol{U}, \boldsymbol{\sigma_U}, \boldsymbol{\theta_{wind}}, \boldsymbol{H_s}, \boldsymbol{T_p}, \boldsymbol{\theta_{wave}})$



Results coming soon

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

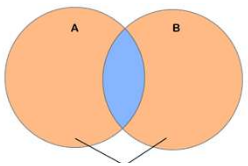
Application to robust active learning of sets

$$\mathcal{O}_{F_n} = \{\mathrm{u}, \mathbb{E}_X\big(F_n(u,X)\big) < s\} \quad \textbf{or} \quad \mathcal{O}_{F_n} = \{\mathrm{u}, \mathbb{P}_X(F_n(u,X) > 0) < s\}$$

$$\boxed{\mathcal{O}_{\mathcal{L}_{F_n}} = \{\mathrm{u}, \mathcal{L}_{F_n}(u) < s\}}$$

where $\mathcal{L}_{F_n}(u) = \mathbb{E}_X\big(F_n(u,X)\big)$ **or** $\mathbb{P}_X(F_n(u,X) > 0)$

- Uncertainty measure : Vorob'ev deviation $d_V(\mathcal{O}_{\mathcal{L}_{F_n}})$



$A\Delta B$

$$\boxed{d_V\left(\mathcal{O}_{\mathcal{L}_{F_n}}\right) = \mathbb{E}_{\mathcal{L}_{F_n}}\left(\lambda\left(\mathcal{O}_{\mathcal{L}_{F_n}} \Delta \mathcal{Q}_{n,\alpha_n^*}\right)\right)}$$

$$\forall u \; \mathcal{L}_{F_n}(u) \sim Law\left(\mu_n^{\mathcal{L}}(u), \sigma_n^{\mathcal{L}}(u)\right)$$

$$\mathcal{Q}_{n,\alpha_n^*} = \{u,\ \mu_n^{\mathcal{L}}(u) + \Phi_{\mathcal{L}}^{-1}(\alpha_n^*)\sigma_n^{\mathcal{L}}(u) \le s\}$$

α_n^* such that $\lambda(\mathcal{Q}_{n,\alpha_n^*}) = \mathbb{E}_{F_n}\left(\lambda(\mathcal{O}_{F_n})\right)$

- SUR criteria on Vorob'ev deviation

$$\boxed{\underset{x}{argmin}\ \mathbb{E}_{\mathcal{L}_{F(x)}|\mathcal{F}_n}\left[d_V\left(\mathcal{O}_{\mathcal{L}_{F_{n+1}}}\right)\right]}$$

POINCARÉ-MALLIAVIN-NUALART INEQUALITIES

Application to robust active learning of sets

where $\mathcal{L}_{F_n}(u) = \begin{cases} \mathbb{E}_X(F_n(u,X)) \\ \textbf{or} \\ \mathbb{P}_X(F_n(u,X) > 0) \end{cases}$

- Inequality on Vorob'ev deviation $d_V(\mathcal{O}_{F_n})$:

$$V_{int} = \int Var(H_u(F_n))du = \int \pi_n(u)(1 - \pi_n(u))du$$

$$H_u(F_n) = \mathbb{I}_{\mathcal{L}_{F_n}(u) < s}$$

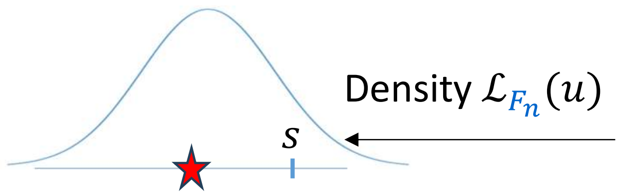
$$\pi_n(u) = \mathbb{P}_{F_n}(\mathcal{L}_{F_n}(u) < s)$$

$$V_{int} \le d_V(\mathcal{O}_{F_n}) \le \frac{V_{int}}{\min(\alpha_n^*, 1 - \alpha_n^*)}$$

- Possible local uncertainty measure

$$Var(H_u(F_n)) \le \mathbb{E}_{F_n}\left(\left\| \mathcal{D} H_u(F_n) \right\|_{TV}\right) = C_{F_n}(u) p_{\mathcal{L}_{F_n}(u)}(s)$$

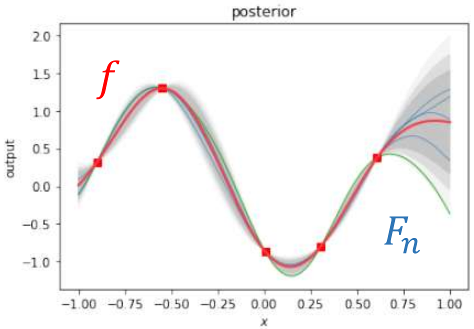
$$V_{int} \le \int C_{F_n}(u) p_{\mathcal{L}_{F_n}(u)}(s) du$$



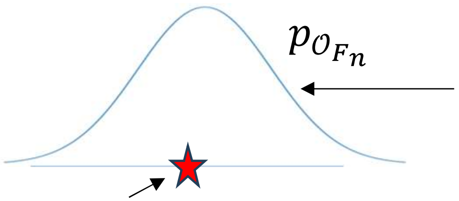
$\mathcal{L}_{F_n}(u) = \mathbb{E}_X(F_n(u,X))$
Random variable

$\mathcal{L}_f(u) = \mathbb{E}_X(f(u,X))$

DOORS ARE WIDE OPEN



$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to \mathbb{R}$



$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X))$
Random variable

$\mathcal{O}_f = \mathbb{E}_X(f(X))$

reduction of the variance $Var(\mathcal{O}_{F_n})$

stepwise uncertainty reduction (**SUR**) approach

$$x_{n+1} = \underset{x}{argmin}\, \mathbb{E}_{Y(x)|\mathbf{Y}_n}\left[Var(\mathcal{O}_{F_{n+1}})\right]$$

$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to L_2(\Omega, \mathbb{R})$

What about

$\mathcal{O}_{F_n} = \mathbb{E}_X(F_n(X)^k)$

$\mathcal{O}_{F_n} = \mathbb{E}_X(g \circ F_n(X))$

$\mathcal{O}_{F_n} = \mathbb{P}_X(g \circ F_n(X) > 0)$

$\mathcal{O}_{F_n} = q_{g \circ F_n(X)}(\alpha)$

$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to L_2(\Omega, \mathbb{R}^d)$

What about

$\mathcal{O}_{F_n} = (S_1, S_2, S_{12}, \dots)$

...

$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to (\Omega, \mathbb{S})$

What about

$\mathcal{O}_{F_n} = \{x, g \circ F_n(x) < s\}$

$\mathcal{O}_{F_n} = \{u, \mathbb{E}_X(g \circ F_n(u, X)) < s\}$

$\mathcal{O}_{F_n} = \{u, \mathbb{P}_X(g \circ F_n(u, X) > 0) < \epsilon\}$

$\mathcal{O}: L_2(\mathbb{R}^d, \Omega) \to L_2(\Omega, \mathbb{H})$

What about

$\mathcal{O}_{F_n} = G(\cdot, F_n)$

...

...

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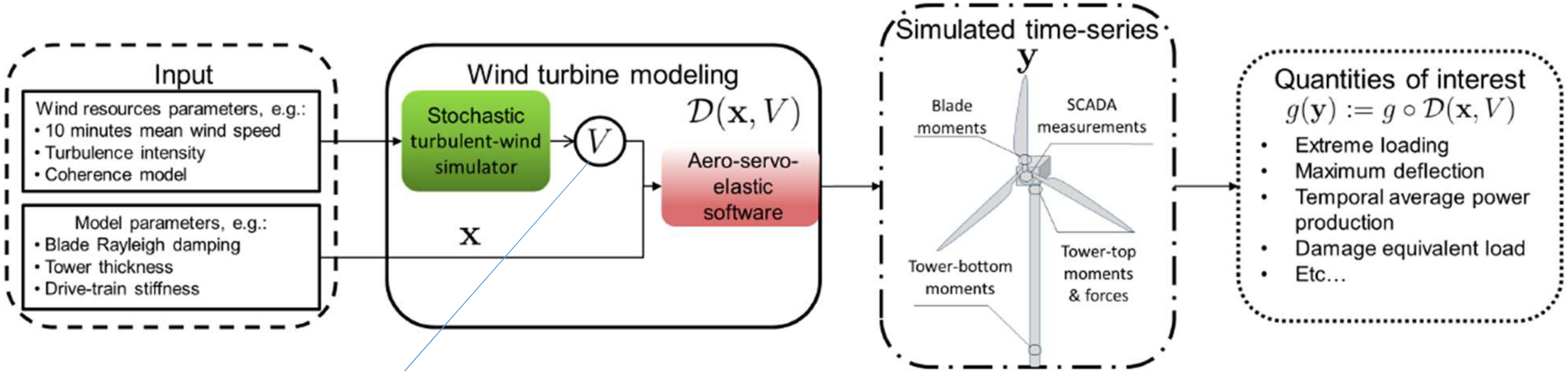
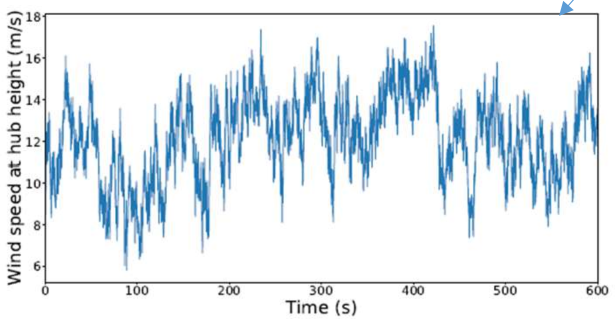
FUNCTIONAL METAMODELLING MOTIVATION





Simulator

$$X \in \mathbb{H} \longrightarrow f \longrightarrow Y \in \mathbb{R}$$

Gaussian process

$$X \in \mathbb{H} \longrightarrow F_n^{\mathcal{O}} \longrightarrow Y \in \mathbb{R}$$

How do we define the Gaussian process with functional inputs: $F_n^{\mathcal{O}}$?

GAUSSIAN PROCESS INDEXED BY FUNCTIONAL INPUTS

- Bounded or unbounded linear GP model such that for $X \in \mathbb{H}$

$$F_n^{\mathcal{O}}(X) = \sum_{i>1} \xi_i \langle X, \mathcal{O}\phi_i \rangle_{\mathbb{H}}$$

  - $\mathcal{O}: \mathbb{V} \to \mathbb{H}$
  - (ϕ_i) orthonormal basis of $\mathbb{V}$
  - (ξ_i) standard normal random variables

TOWARDS OPERATOR BASED FUNCTIONAL-RELIABILITY

$$\mathbb{P}_{F_{in}}\left(\max_{t\in[O,T]} F_n^{\mathcal{O}}(t; F_{in}) > g_{max}\right) \sim 10^{-4}$$

- Space-filling design for functional inputs: $\mathcal{X}_0$ *[Thesis Calzolari, Nov. 2024 - Nov. 2027]*
- Building a GP model with functional inputs associated to scalar or time dependent output
- Probability estimation with active learning …

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