



# Analysis of single-ring infiltrometer data for soil hydraulic properties estimation: Comparison of BEST and Wu methods

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## ABSTRACT

Knowledge of soil hydraulic properties is important for modeling hydrological processes and related contaminant transport. This study compared four methods in analyzing single-ring infiltrometer data to estimate the saturated hydraulic conductivity ( $K_s$ ) and the water retention parameter ( $\alpha$ ). These were: (1) original BEST (Beerkan Estimates of Soil Transfer Parameters through Infiltration Experiments, Lassabatere et al., 2006) method, defined as BEST\_slope; (2) a modified BEST method, defined as BEST\_intercept (Yilmaz et al., 2010); (3) Wu1 (Wu et al., 1999) which attempts the best fit of a generalized solution to the infiltration curve using the whole infiltration curve; and (4) Wu2 (Wu et al., 1999) which is suitable for the steady state flow case. The first three methods are suitable for the transient flow state. The infiltration data of 54 different cases within four soil texture classes (sand, sandy loam, medium loam, and clay loam) were used. The results suggest that the modified version (BEST\_intercept) has a better performance (more reasonable estimates) than the original (BEST\_slope). Both the BEST\_slope and BEST\_intercept methods perform poorly for the sandy soils. The Wu1 method performs better in fitting the experimental infiltration curve, and produces more cases with reasonable values (normally positive values) of  $K_s$  and  $\alpha$  than both the BEST\_slope and BEST\_intercept. In order to apply these existing methods to wider conditions (e.g., sandy soils, wet soils, basic oxygen furnace slag), the inversion estimation algorithms and the experimental operations in the field require further improvement.

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## 1. Introduction

Knowledge of soil hydraulic properties (e.g., saturated hydraulic conductivity,  $K_s$  and water retention parameter,  $\alpha$ ) is important for modeling hydrological processes and related contaminant transport. These properties vary very much with soils, time and space (Mubarak et al., 2009, 2010; Schaap et al., 2001). High accuracy of measurements and estimation of these properties still remain challenges. Measurements of soil hydraulic properties can be conducted either in the laboratory or in the field using different methods. The methods with minimum soil disturbance, low time consumption and lowest cost are preferred. The Beerkan method (Haverkamp et al., 1996) includes a single-ring infiltration field measurement using a metal or PVC ring inserted into initially unsaturated soils

to a given small depth, and appears promising due to its ease of operation and low cost. Furthermore, several studies have promoted its robustness by introducing new algorithms (Braud et al., 2005; Lassabatere et al., 2006, 2009; Yilmaz et al., 2010). Among them is the BEST (Beerkan Estimates of Soil Transfer Parameters through Infiltration Experiments) method. The original method is known as the BEST\_slope and the modified version is called the BEST\_intercept following Yilmaz et al. (2010). The BEST method is based on the van Genuchten relationship (van Genuchten, 1980) for the water retention curve:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = [1 + (\alpha h)^n]^{-m} \quad (1a)$$

with the Burdine condition (Burdine, 1953),

$$m = 1 - \frac{2}{n} \quad (1b)$$

and the Brooks and Corey relationship (Brooks and Corey, 1964) for hydraulic conductivity:

$$\frac{K(\theta)}{K_s} = \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^\eta \quad (2a)$$

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with

$$\eta = \frac{2}{\lambda} + 2 + p \quad \text{and} \quad \lambda = mn \quad (2b)$$

where  $n$ ,  $m$  and  $\eta$  are shape parameters;  $\alpha$  (water retention parameter),  $\theta_r$  (residual volumetric soil water content),  $\theta_s$  (saturated volumetric soil water content), and  $K_s$  (saturated hydraulic conductivity) are scale parameters;  $\theta$  and  $K(\theta)$  are soil water content and hydraulic conductivity at unsaturated state, respectively;  $h$  is the water pressure head. Usually,  $\theta_r$  is very low and thus considered to be zero.  $p$  is a tortuosity parameter that depends on the chosen capillary model, and a value of 1 is used here following Burdine's condition (Braud et al., 2005; Burdine, 1953).

The BEST (BEST\_slope) method performed better than other analysis methods (cumulative linearization, derivative linearization, cumulative infiltration, and infiltration flux) using the same experimental infiltration data (Lassabatere et al., 2006; Xu et al., 2009) in that it (BEST\_slope) produced more reasonable results. But there still remain some problems (the occurrence of null or even negative estimates of  $K_s$ ) with the BEST\_slope method as noted by Lassabatere et al. (2010), Xu et al. (2009) and Yilmaz et al. (2010). Recently, Yilmaz et al. (2010) proposed a modified version (BEST\_intercept). The modified version solved the problems that the original version does not work or produce some unreasonable estimates (negative  $K_s$ ) under certain conditions. More experiments are therefore required to further test the performance and application of BEST and its modified version, and to search for answers to the remaining problems. Previous studies (Lassabatere et al., 2006; Xu et al., 2009) have shown a better performance of the BEST method relative to other methods (stated above). Hence, this study compares the BEST with another method, namely the Wu method (Wu et al., 1999). The latter was developed to calculate  $K_s$  by best fit of a generalized solution to the infiltration curve of single-ring infiltrometer data. The first method in Wu, hereafter named Wu1, uses the whole infiltration curve and the second, hereafter named Wu2, is based on the assumption that over the last part of the infiltration curve, the event has reached steady state. Bagarello et al. (2009) have shown that the Wu method was reliable in estimating both  $K_s$  and  $\alpha$  with single-ring infiltrometer data from an Italian study. The Wu analysis method seems also applicable to the infiltration data from Beerkan infiltration experiments.

This study therefore aims to: (1) provide more tests on the performance and application of the BEST method, and to explain any remaining problems; and (2) compare the performance of the BEST and Wu methods in analyzing the same single-ring infiltration data to estimate the soil hydraulic properties,  $K_s$  and  $\alpha$ .

## 2. Theory

### 2.1. BEST (Lassabatere et al., 2006)

Considering an infiltration experiment with zero pressure on an internal-radius  $r$  of a circular cylindrical surface above a uniform soil with a uniform initial soil water content, the three-dimensional cumulative infiltration and steady infiltration rate can be approached by the explicit transient two-term equation:

$$I(t) = S\sqrt{t} + (ES^2 + FK_s)t \quad (3a)$$

and steady-state expansion:

$$I_{+\infty}(t) = (ES^2 + K_s)t + G\frac{S^2}{K_s} \quad (3b)$$

$$q_s = ES^2 + K_s \quad (3c)$$

where  $t$  is time,  $I(t)$  is cumulative infiltration at transient state,  $I_{+\infty}$  is cumulative infiltration at steady state,  $q_s$  is steady infiltration rate,

and  $S$  is sorptivity, respectively; constants  $E$ ,  $F$ , and  $G$  are defined by Haverkamp et al. (1994) as:

$$E = \frac{\gamma}{r(\theta_s - \theta_0)} \quad (4a)$$

$$F = \frac{2 - \beta}{3} \left[ 1 - \left( \frac{\theta_0}{\theta_s} \right)^\eta \right] + \left( \frac{\theta_0}{\theta_s} \right)^\eta \quad (4b)$$

$$G = \frac{1}{2[1 - (\theta_0/\theta_s)^\eta](1 - \beta)} \ln \left( \frac{1}{\beta} \right) \quad (4c)$$

where  $r$ ,  $\theta_0$  and  $\theta_s$  are the internal radius of cylindrical ring used, initial and saturated volumetric soil water content, respectively;  $\beta = 0.6$  and  $\gamma = 0.75$ , which apply for most soils when  $\theta_0 < 0.25 \theta_s$  (Haverkamp et al., 1994; Smettem et al., 1994);  $\eta$  is a shape parameter that can be estimated (Eq. (2b)) from particle size distribution and soil porosity (for details see Lassabatere et al., 2006).

BEST first estimates the apparent steady-state infiltration rate ( $q_s$ ) through fitting the last part of the infiltration curve (cumulative infiltration vs. time). Then BEST estimates the sorptivity ( $S$ ) by fitting the transient cumulative infiltration to the two-term equation, Eq. (3a). The fit is based on the replacement of hydraulic conductivity  $K_s$  by its sorptivity function  $S$  and the experimental apparent steady-state infiltration rate ( $q_s$ ) through Eq. (3c) and the following conditions: an accurate reproduction of experimental data; a fit for  $S$  between zero and a maximum value that corresponds to a null hydraulic conductivity (capillary driven flow). Once sorptivity is estimated, the saturated hydraulic conductivity is obtained through Eq. (3c), assuming that the apparent steady state has been reached. In order to ensure the validity of Eq. (3a), the data subsets used should be restricted within the maximum time ( $t_{\max}$ ), defined in Lassabatere et al. (2006) as:

$$t_{\max} = \frac{1}{4(1 - F)^2} t_{\text{grav}} \quad (5a)$$

$$t_{\text{grav}} = \left( \frac{S}{K_s} \right)^2 \quad (5b)$$

The parameter  $\alpha$  is then estimated from the other hydraulic parameters (Haverkamp et al., 2006; Lassabatere et al., 2006):

$$\alpha = \frac{c_p \theta_s (1 - (\theta_0/\theta_s)) K_s [1 - (\theta_0/\theta_s)^\eta]}{S^2} = \frac{c_p (\theta_s - \theta_0) (K_s - K_0)}{S^2} \quad (6a)$$

$$c_p = \Gamma(1 + (1/n)) \left\{ \frac{\Gamma(m\eta - (1/n))}{\Gamma(m\eta)} + \frac{\Gamma(m\eta + m - (1/n))}{\Gamma(m\eta + m)} \right\} \quad (6b)$$

where  $\Gamma$  is the usual Gamma function, and  $K_0$  is the initial hydraulic conductivity calculated by Eq. (2);  $n$ ,  $m$  and  $\eta$  can be estimated from particle size distribution and soil porosity (see the details in Lassabatere et al., 2006).

Yilmaz et al. (2010) found that there are some problems in BEST estimates (the occurrence of null or even negative estimates of  $K_s$ ) under the condition of  $q_s \sim ES^2$ . They therefore modified the original BEST method by using the intercept ( $b_{+\infty}$ ) of the asymptotic expansion  $I_{+\infty}(t)$ , as defined in Eq. (3b), to estimate  $K_s$ :

$$K_s = G \frac{S^2}{b_{+\infty}} \quad (7)$$

Other than this, the calculation procedure is the same as the original version (details see Yilmaz et al., 2010). In this study, the original version is called BEST\_slope, and the modified version is called BEST\_intercept.

## 2.2. Wu method (Wu et al., 1999)

The Wu1 method is based on the assumption that the cumulative infiltration curve can be used to describe the infiltration process:

$$I = At + Bt^{0.5} \quad (8)$$

where  $I$  is cumulative infiltration,  $t$  is time, and  $A$  and  $B$  are empirical coefficients. This equation is fitted to the  $(t, I)$  data pairs measured from the beginning of the single-ring infiltration experiments to obtain an estimate of  $A$  and  $B$ . Then  $K_s$  is calculated by the following equation:

$$K_s = \frac{\Delta\theta \left[ \sqrt{(H + G^*)^2 + 4G^*C} - (H + G^*) \right]}{2T_c} \quad (9)$$

where  $\Delta\theta$  is the difference between the saturated volumetric soil water content ( $\theta_s$ ), and the initial volumetric soil water content ( $\theta_0$ );  $H$  is the ponded depth in the ring; the  $G^*$ ,  $C$  and  $T_c$  terms have the following expressions:

$$G^* = d + \frac{r}{2} \quad (10a)$$

$$C = \frac{1}{4\Delta\theta} \left( \frac{B}{b} \right)^2 \frac{a}{A} \quad (10b)$$

$$T_c = \frac{1}{4} \left( \frac{Ba}{bA} \right)^2 \quad (10c)$$

where  $d$  and  $r$  are the insert depth of the ring and the ring radius, respectively, and  $a$  and  $b$  are dimensionless constants ( $a=0.9084$ , and  $b=0.1682$ ). An estimate of the  $\alpha$  parameter can be obtained by the following relationship:

$$\alpha = \frac{K_s}{\phi_m} \approx \frac{K_s}{\phi'_m} = \lambda_c^{-1} \quad (11a)$$

where  $\phi'_m$  is given by:

$$\phi'_m = \frac{K_s^2 T_c}{\Delta\theta} \quad (11b)$$

$\lambda_c$  is the capillary length that represents the importance of capillary forces relative to gravity for water movement (White and Sully, 1987).

Note that the  $\alpha$  parameter in Wu is from Gardner function (Gardner, 1958), and is different from that of van Genuchten function used in BEST (Eq. (1a)). However, Haverkamp et al. (2006) presented a generalized form of the capillary length that equals the reciprocal of the right-hand side of Eq. (6a). Therefore the  $\alpha$  parameter in Wu and that in BEST are linked with each other via the capillary length. According to Mubarak et al. (2010), when describing unsaturated water flow subject to a given set of initial and boundary conditions, the water flow behaviour (either capillary forces or gravity dominant) of the soil should be independent of the choice of the soil hydraulic functional relationships. Thus the  $\alpha$  parameter in Wu and that in BEST can be directly compared with each other.

The Wu2 method makes the assumption that steady state has been reached during the infiltration test. Then it is possible to fit the infiltration curve (the last part of the curve) with a linear equation:

$$I = it + c = afK_s t + c \quad (12)$$

where  $I$  is the cumulative infiltration with time  $t$ ,  $i$  is the slope of the infiltration curve (a steady state infiltration rate),  $a$  is a dimensionless constant =0.9084 and  $f$  is a correction factor depending on soil and ring geometry. The correction factor  $f$  can be estimated by:

$$f = \frac{H + (1/\alpha)}{G^*} \quad (13)$$

where  $H$  is the ponded depth in the ring,  $G^*$  is from Eq. (10a) and  $\alpha$  is a factor depending on the soil texture class ( $\alpha=0.36 \text{ cm}^{-1}$  for sand,  $\alpha=0.12 \text{ cm}^{-1}$  for medium loam and  $\alpha=0.04 \text{ cm}^{-1}$  for clay, see Wu et al., 1999).

The saturated soil hydraulic conductivity is then calculated by:

$$K_s = \frac{i}{af} \quad (14)$$

## 3. Materials and methods

### 3.1. Field experiments

Twenty sites of different mineral soil types, throughout Ireland, with grassland land cover were selected to conduct the single-ring field infiltration experiments. Heavy duty plastic rings with a 14.4 cm internal diameter (of wall thickness ~4 mm) with the driving edge bevelled (on the external side of the ring) were used. Lassabatere et al. (2006) used ring diameters of 15 and 20 cm, and Mubarak et al. (2010) used a ring diameter of 13 cm. Our ring diameter of 14.4 cm is in this range. The soil hydrological properties determined from this study will be used to run a process based catchment hydrology model known as GEOtop (Rigon et al., 2006). This model requires these soil parameters not only at the surface but also for a number of subsoil layers. The field tests were therefore carried out at three depths, the surface, 15 and 30 cm (with three replicates each) for each site. During the field experiments for subsoils, the upper soils were removed to allow sufficient space to prevent suction gradient induced upward infiltration at the ring perimeter.

The Beerkan field infiltration method (Lassabatere et al., 2006) used in this study, is a simple three-dimensional infiltration test under positive head,  $h_{sur}$  conditions (the head at the soil surface) using a cylinder of known diameter. The procedure is carried out in consecutive steps as follows. The surface vegetation is removed over an area slightly larger than the cylinder diameter while the roots remain in situ. The test is made on a level site. The cylinder is positioned at the soil surface and inserted to a depth no deeper than 1 cm into the topsoil to prevent lateral losses of water. A soil sample is collected (0–5 cm depth) close to but not adjacent to the cylinder, and used to determine the initial gravimetric water content. At the end of the experiments, the saturated soil is sampled to determine the saturated gravimetric water content. Particle-size distribution (soil texture information) analysis is carried out on another soil sample taken near the cylinder at the experimental site. A fixed volume of water (exactly 178 ml for the 14.4 cm diameter ring corresponding to a water depth of 1.1 cm in this study) is poured into the cylinder at time zero, and the time required for infiltration of the known volume of water is measured. As soon as the first volume has completely infiltrated, i.e. water no longer standing on the soil surface, the second known volume of water (also 178 ml) is added to the cylinder and the time is recorded for this to infiltrate (cumulative time). The procedure is repeated until the test reaches nearly steady-state (apparent steady-state). This is reached when three consecutive infiltration times are identical, and the cumulative infiltration is then recorded (Lassabatere et al., 2006; Mubarak et al., 2010). With this procedure, the surface pressure is not constant during the test (the “falling head” test). However, Haverkamp et al. (1998) have shown that small variations of  $h_{sur}$  do not significantly influence the results. After the experiment, an undisturbed sample of known volume (soil core) is taken close to the cylinder to obtain the soil dry bulk density,  $\rho_d$  ( $\text{g cm}^{-3}$ ) and then porosity (set particle density as  $2.65 \text{ g cm}^{-3}$ ). The initial and saturated volumetric water contents were estimated using:

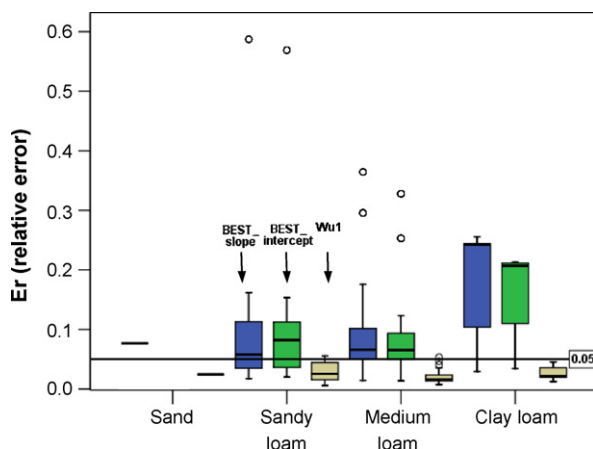
$$\theta = \frac{w\rho_d}{\rho_w} \quad (15)$$

**Table 1**  
Basic soil physical properties of the cases used in this study.

Texture (N)	Variable	Sand (%)	Silt (%)	Clay (%)	Bulk density (g cm <sup>-3</sup> )	Porosity	Initial soil water content	Saturated soil water content	Ratio of porosity to saturated water content
Sand (9)	Minimum	92.3	0.1	2.0	0.79	0.51	0.09	0.26	1.4
	Maximum	97.8	2.4	5.5	1.29	0.70	0.22	0.51	2.1
	Mean	96.2	0.8	3.0	1.13	0.58	0.14	0.34	1.7
	Std. dev	2.0	0.9	1.4	0.25	0.09	0.04	0.07	0.2
Sandy loam (11)	Minimum	50.9	22.6	4.5	0.64	0.55	0.13	0.32	1.0
	Maximum	73.0	42.6	17.7	1.19	0.76	0.45	0.73	1.8
	Mean	58.3	31.6	10.1	0.98	0.63	0.33	0.52	1.3
	Std. dev	7.1	6.1	5.0	0.22	0.08	0.10	0.12	0.3
Medium loam (29)	Minimum	34.7	27.8	10.0	0.90	0.37	0.12	0.19	1.0
	Maximum	52.1	48.9	26.4	1.66	0.66	0.36	0.53	3.0
	Mean	43.7	37.8	18.6	1.13	0.57	0.25	0.35	1.8
	Std. dev	5.3	5.9	4.4	0.17	0.06	0.06	0.08	0.5
Clay loam (5)	Minimum	23.3	31.9	28.6	1.07	0.52	0.13	0.19	1.9
	Maximum	38.2	41.2	35.5	1.26	0.60	0.29	0.32	3.0
	Mean	31.8	36.7	31.5	1.14	0.57	0.22	0.27	2.2
	Std. dev	5.4	4.4	3.6	0.08	0.03	0.06	0.05	0.5

Std. dev. denotes standard deviation.

where  $\theta$  is the initial or saturated volumetric water content,  $w$  is the corresponding initial or saturated gravimetric water content, and  $\rho_w$  is the specific density of water ( $\rho_w = 1 \text{ g cm}^{-3}$ ). The saturated soil samples are collected by a ring with a height of 5 cm, but the infiltrated water reaches less than 5 cm depth at the end of infiltration experiments. It means that the so-called saturated sample includes two parts: saturated upper part and unsaturated lower part. This results in an underestimation of the saturated water content. This is why high ratios of porosity to volumetric saturated water content are found in this study (Table 1). We therefore use the porosity to replace the volumetric saturated water content when using these methods (BEST and Wu) in this study, as done by many other studies (Lassabatere et al., 2010; Mubarak et al., 2010; Yilmaz et al., 2010). The particle size distribution (PSD) is measured following ASTM F1632-03 (2010) with silt and clay fractions measured by the pipet method but coarse fractions measured by mechanical sieving. There are eight particle size limits (1–2 mm, 0.5–1 mm, 0.25–0.5 mm, 0.15–0.25 mm, 0.106–0.15 mm, 0.053–0.106 mm, 0.002–0.053 mm and 0–0.002 mm) for each sample. The measured PSD information and porosity are then used to estimate shape parameters ( $n$ ,  $m$  and  $\eta$ ) following Lassabatere et al. (2006). Most (43/54) of the relative errors between the simulated and the measured PSD curves are less than 0.05, and therefore the PSD fitting method (Lassabatere et al., 2006) is reliable.



**Fig. 1.** Boxplot of relative errors for each method by soil texture class.

In total, 180 cases (20 sites  $\times$  3 depths  $\times$  3 replicates) were examined in these field experiments. Of these, 54 cases of 4 soil texture classes (see Table 1) were selected for analysis that satisfied the following criterion:

- 1) At least 7 data points ( $t, I$ ) in one infiltration process (to ensure meeting the requirement of BEST – at least 5 points at transient state);
- 2) Available datasets of soil water content (to ensure the condition of soil porosity  $> \theta_s > \theta_0$ );
- 3) Available datasets of soil particle size distribution (PSD);
- 4) Insert depth of ring recorded;
- 5) No exception in the infiltration processes (e.g., no rain or leaking recorded);
- 6) At least 3 cases for each soil texture class.

### 3.2. Statistical analysis

The relative error ( $Er$ ) was used to assess how well the curve fit from each fitting method (BEST and Wu1) match with the experimental infiltration process (Lassabatere et al., 2006).  $Er$  is defined as:

$$Er = \sqrt{\frac{\sum_{i=1}^k (y_i^{\text{exp}} - y_i)^2}{\sum_{i=1}^k (y_i^{\text{exp}})^2}} \quad (16)$$

where  $y_i^{\text{exp}}$  ( $i = 1 \dots k$ ) are the experimental data, and  $y_i$  ( $i = 1 \dots k$ ) are the correspondingly fitted data.

A lower  $Er$  represents better performance of the fitting method (BEST or Wu1). The nonlinear least square fitting analysis and the  $Er$  calculation for both BEST and Wu1 were coded in MATLAB R2007b. Statistical analysis (e.g., basic statistics, correlation analysis) was conducted with SPSS 15.0 software.

## 4. Results

### 4.1. Descriptions of the application of the four methods

For the relative error ( $Er$ ) on fitting the experimental infiltration data, the Wu1 method was found to have lower values than the BEST method (both BEST\_slope and BEST\_intercept) in each soil texture (Fig. 1). All of the  $Er$  values were found to be less than 6%

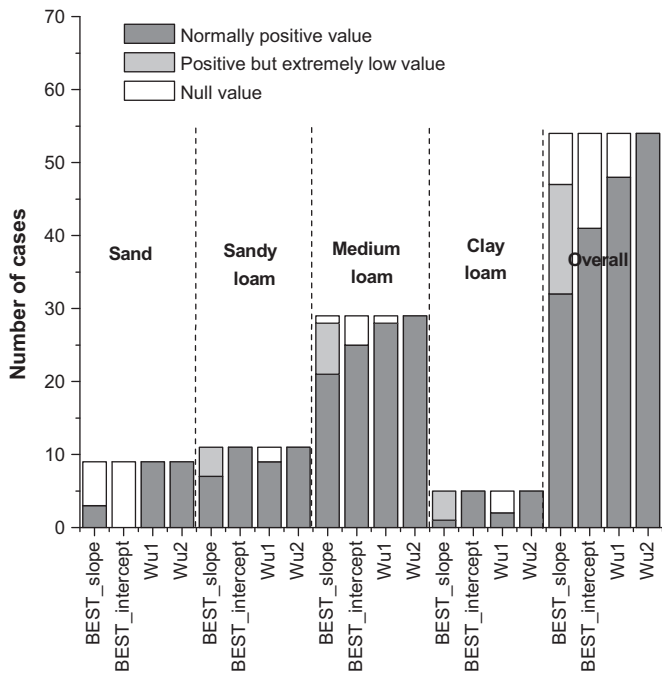


Fig. 2. Number of cases with normally positive, positive but extremely low values, null values of  $K_s$  and  $\alpha$  for each method within each soil texture class.

for the Wu1 method (Fig. 1), and actually most (51/53) of them were less than 5%. For the BEST\_slope method, only 1 of 3 cases in sand, 5 of 11 cases in sandy loam, 7 of 28 cases in medium loam class and 1 of 5 cases in clay loam class had  $Er$  values less than 5%. The BEST\_intercept method had a similar performance to the BEST\_slope method (Fig. 1).

Of the four methods: BEST\_slope was unable to calculate  $K_s$  and  $\alpha$  value for 6 of the 9 cases in sand, 2 of 11 cases in sandy loam, and 1 of 29 cases in medium loam. In the cases with positive values of  $K_s$  and  $\alpha$ : 4 of 11 cases in sandy loam, 7 of 28 cases in medium loam and 4 of 5 cases in clay loam resulted in extremely low values of  $K_s$  and  $\alpha$  (Fig. 2). These extremely low values of  $K_s$  and  $\alpha$  are at  $10^{-8}$  cm day $^{-1}$  and  $10^{-10}$  cm $^{-1}$  orders of magnitude (see Table 2). The values of  $K_s$  are 9 or 10 orders of magnitude lower than corresponding steady state infiltration rates. BEST\_intercept was unable to calculate  $K_s$  and  $\alpha$  value for all of the cases (9 of 9) in sand, 4 of 29 cases in medium loam, while it normally works for every case in sandy loam and clay loam. The Wu1 method was unable to calculate  $K_s$  and  $\alpha$  value for 2 of 11 cases in sandy loam, 1 of 29 cases in medium loam, and 3 of 5 cases in clay loam, but no extremely low values of  $K_s$  and  $\alpha$  were found using the Wu1 method. The Wu2 (only used to calculate  $K_s$ ) method was found to give normally positive  $K_s$  for every case in each texture class.

#### 4.2. Comparison of $K_s$ and $\alpha$ among different methods

Among the four soil texture classes, the sand class has the highest values for  $K_s$  and  $\alpha$ , followed by sandy loam, medium loam and clay loam, respectively (Table 2 and Fig. 3). The four methods correlated well with each other in the estimates of  $K_s$  and  $\alpha$  (Table 3). Of the four methods, BEST\_intercept and Wu2 method resulted in higher values of  $K_s$  (Fig. 3a) and  $\alpha$  (Fig. 3b) than BEST\_slope and Wu1. The BEST\_slope estimates were more comparable to the Wu1 estimates, while the BEST\_intercept were more comparable to the Wu2 estimates (Fig. 3).

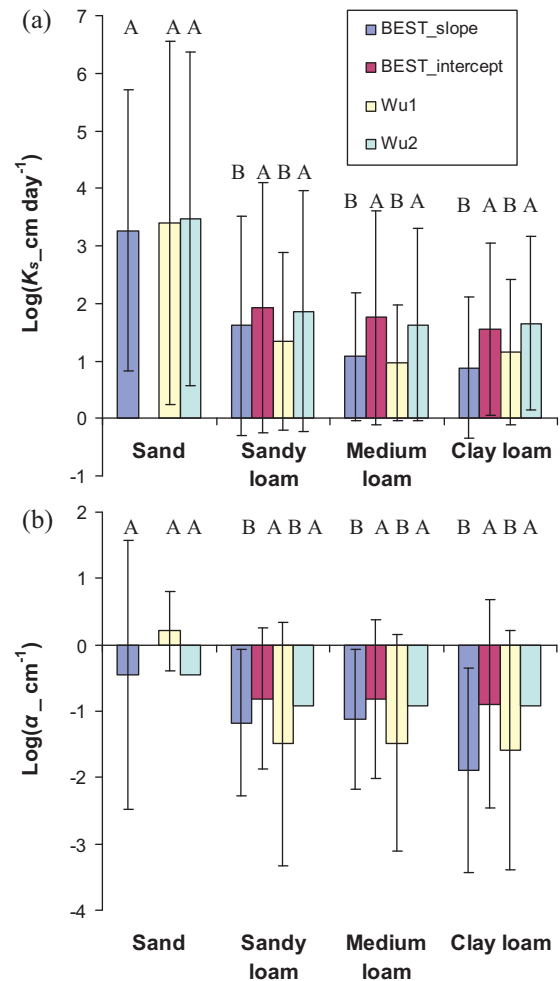


Fig. 3. Comparison of the values of (a)  $K_s$  and (b)  $\alpha$  among different methods. Note that different letters (A, B) represent significant difference between any two groups at 0.05 significance level made by non-parametric tests (Mann–Whitney U method).

## 5. Discussion

### 5.1. Why do null and extremely low values of $K_s$ and $\alpha$ occur for some cases?

The BEST method (both BEST\_slope and BEST\_intercept) performed poorly for the sand class (Fig. 2), in which the apparent steady flow state is quickly reached resulting in many data points at steady state but insufficient data points for the transient flow state to satisfy the method (an example of infiltration process is shown in Fig. 4a). In BEST, at least 5 points at the transient flow state are required to fit the experimental data to Eq. (3a). Xu et al. (2009) has also identified this aspect. In another study for the urban infiltration basin, Lassabatere et al. (2010) also found the BEST\_slope method did not work for subsoil (sandy soil) because of the lack of convexity at the beginning of the cumulative infiltration curve.

Another problem with the BEST\_slope method is the occurrence of extremely low  $K_s$  and  $\alpha$  for some cases (Fig. 2). According to Eq. (3c), this situation occurs when  $ES^2$  is very close to  $q_s$ . Applying BEST (BEST\_slope) to a new urban soil (basic oxygen furnace slag), Yilmaz et al. (2010) found that when the estimated  $ES^2$  value exceeds the infiltration rate at the end of the experiment, the values obtained for  $K_s$  are even negative. They then modified the BEST\_slope to the BEST\_intercept method. This newly modified method produced more reasonable values than the original BEST in this study (Table 2 and Fig. 2).

**Table 2**  
Statistics of estimated hydraulic properties for different soil texture classes.

Texture	Parameter	Method	N	Min	Max	Mean	Std. dev.	CV (%)
Sand	$K_s$ (cm day <sup>-1</sup> )	BEST_slope	3	1536.9	2070.4	1847.5	277.3	15
		BEST_intercept						
		Wu1	9	824.9	4573.5	2516.8	1434.3	57
	$\alpha$ (cm <sup>-1</sup> )	Wu2	9	1645.6	4014.1	2928.4	779.9	27
		BEST_slope	3	0.3444	0.3629	0.3527	0.0094	3
		BEST_intercept						
Sandy loam	$K_s$ (cm day <sup>-1</sup> )	Wu1	9	0.0852	12.1978	1.6231	3.9734	245
		Wu2	9	0.36	0.36	0.36	0	0
		BEST_slope	11	1.30E-08	216.1	40.9	80.1	196
	$\alpha$ (cm <sup>-1</sup> )	BEST_intercept	11	3	431.1	83.3	152	183
		Wu1	9	0.5	97.1	21.5	34.9	162
		Wu2	11	6.1	354.1	72.6	123.5	170
Medium loam	$K_s$ (cm day <sup>-1</sup> )	BEST_slope	11	1.00E-10	0.2313	0.0666	0.0778	117
		BEST_intercept	11	0.0598	0.3463	0.1546	0.0871	56
		Wu1	9	0.0075	0.0457	0.032	0.0148	46
	$\alpha$ (cm <sup>-1</sup> )	Wu2	11	0.12	0.12	0.12	0	0
		BEST_slope	28	1.70E-08	51	11.8	12.9	109
		BEST_intercept	25	13.2	344.7	56.8	71.8	126
Clay loam	$K_s$ (cm day <sup>-1</sup> )	Wu1	28	0.5	43	9.4	10.1	107
		Wu2	29	6.5	249.5	42.3	47.2	112
		BEST_slope	28	3.60E-11	0.2861	0.0756	0.0876	116
	$\alpha$ (cm <sup>-1</sup> )	BEST_intercept	25	0.0634	0.3386	0.1514	0.0627	41
		Wu1	28	0.0099	0.1125	0.0329	0.0232	71
		Wu2	29	0.12	0.12	0.12	0	0

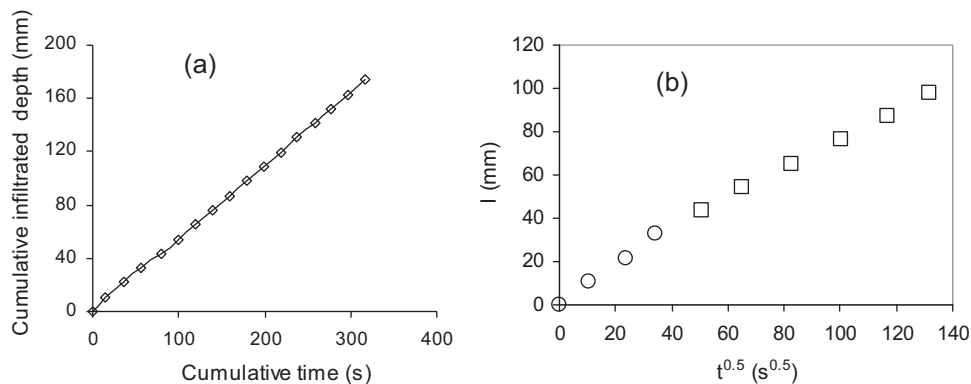
Std. dev. and CV denote standard deviation and coefficients of variation, respectively.

**Table 3**  
Pearson correlation coefficients for  $K_s$  and  $\alpha$  between different methods.

	$K_s$ _slope	$K_s$ _intercept	$K_s$ _Wu1	$K_s$ _Wu2	$\alpha$ _slope	$\alpha$ _intercept	$\alpha$ _Wu1
$K_s$ _slope	1	0.795**	0.990**	0.996**			
$K_s$ _intercept		1	0.745**	0.808**			
$K_s$ _Wu1			1	0.935**			
$K_s$ _Wu2				1			
$\alpha$ _slope					1	0.905**	0.745**
$\alpha$ _intercept						1	0.298

\*Significance level at 0.05.

\*\* Significance level at 0.001.



**Fig. 4.** Examples of exceptional cases (a) many data points at steady state but insufficient data points at transient flow state, and (b) two different parts presented in one infiltration process with the first part (circles) showing steeper than the second (squares).

Another question may also arise. The  $\beta$  and  $\gamma$  values used in this investigation (0.6 and 0.75, respectively) apply for initially dry soil conditions ( $\theta_0 < 0.25 \theta_s$ ) as stated above, but the condition of  $\theta_0 > 0.3 \theta_s$  was common in this investigation (Table 1). It seems the assumption ( $\theta_0 < 0.25 \theta_s$ ) was violated by this wet condition. However, the recent study by Lassabatere et al. (2009), based on a numerical simulation, found that the values of  $\beta$  and  $\gamma$  scarcely depend upon the initial degree of saturation. But we should note that the constraint of  $\theta_0 < 0.25 \theta_s$  is also used to ensure a proper convexity of  $I(t)$  curve. This convexity is required for proper fits of all transient models. This is why the transient models, BEST\_slope and BEST\_intercept, conducted a poor performance in fitting the infiltration curve (Fig. 1) for the wet conditions ( $\theta_0 > 0.3 \theta_s$  frequently occurred in Ireland).

For all of the cases where the Wu1 method did not work (i.e. producing null values), the BEST\_slope produced extremely low values. For these cases, the  $A$  coefficient in Eq. (8) is negative and its absolute value is low enough (with absolute value of  $C$  in Eq. (10b) being high enough) to cause  $(H+G^*)^2 + 4G^*C$  to be negative, and thereby Eq. (9) does not work. Moreover, no such cases predominate in the sand class. Examining the infiltration curves (cumulative infiltration vs. square root of time) of these cases, we found the curves always present two very contrasting parts (an example in Fig. 4b): the first (earlier) part is significantly steeper than the second (later) one, which changes the parabolic curve from upward (for valid cases) to downward (for invalid cases) with  $A$  in Eq. (8) being negative. It indicates the succession of two different flow regimes corresponding to water infiltration in a layered profile, with a first infiltration into the top layer followed by another infiltration into the subsoil. Such layering could be explained by the step of soil preparation before water infiltration experiments. We may have disturbed the soil, in the procedure used to clear the grass vegetation (for surface soils, the first layer) or in our procedure of leveling off the top end (for the second layer-15 cm or third layer-30 cm) using the scissors or scoop. This may leave some loose soil near the top end of that layer (surface, 15 or 30 cm) possibly resulting in higher porosity relative to the lower part of that layer. This may lead to higher infiltration rates at the beginning of the experiments but lower infiltration once the porosity of the top end of that layer becomes saturated. Another reason may be that the perimeter of the infiltration ring after installation in the soil was undetectably loose, enabling water leakage around the perimeter ring at the early phase, although we did not notice it in the field and made no such records in our fieldbook.

## 5.2. Comparison of the BEST and Wu methods in analyzing the infiltration data

How well the fitted curves match with the observed (relative error,  $Er$ ) is one of the more important indicators of performance of the fitting methods (BEST and Wu1). Most (51/53) of the  $Er$  values of Wu1 are lower than 5% and it is also lower than the  $Er$  of BEST for each soil texture class (Fig. 1). It suggests that the shape of the fitted infiltration curve with the parameters from Wu1 is closer to the experimental infiltration curve than the fitted curve from BEST. We should also note that the Wu1 estimation is based on 2 degrees of freedom for the fit (optimization of  $A$  and  $B$  in Eq. (8)) instead of only 1 degree of freedom for the BEST algorithm (optimization of sorptivity  $S$  in Eq. (3a),  $K_s$  being derived from  $S$ ). This may explain the better fits obtained when using Eq. (8) since fitting is better when the degree of freedom increases.

However, the fit quality is not the only criterion to be considered in validating a model. Importance needs to be given to the physical meaning of model parameters when comparing two models. The two equations (Eq. (8)) and (Eq. (3a)) were not obtained in the same way. Eq. (3a) was deduced from the analytical integration of Richards equation (Richards, 1931) assuming uniform initial

water content proposed by Haverkamp et al. (1994) and the related parameters ( $S$ ,  $E$ ,  $F$  and  $K_s$ ) have a physical meaning. Eq. (8) is based on the numerical solution of Richards equation using a specific scaling procedure (Wu et al., 1997) and the fit of the numerically generated flow rate to an empirical law,  $a + b/t$ . Then the related parameters,  $A$  and  $B$ , are derived from several semi-empirical relations involving the fitted constants ( $a = 0.9084$  and  $b = 0.1642$ ) and defined through relations with no clear physical meaning.

As for the outputs of  $K_s$  and  $\alpha$ , the estimates of the BEST and Wu methods are well related with each other (Table 3). This is expected because common starting data is used for all of the methods. But the outputs of  $K_s$  and  $\alpha$  regroup the four methods into two classes: BEST\_slope and Wu1 versus BEST\_intercept and Wu2 (Fig. 3). The  $K_s$  and  $\alpha$  decrease from sandy soils to clayey soils (Fig. 3). This suggests that the estimates of all the methods are reasonable. Soil texture is a good criterion in distinguishing soils in regard to soil hydraulic properties (e.g., used by Schaap et al., 2001), but we should note that soil structure (macropores, fractures, etc.) also play an important role in controlling hydraulic behaviors of soils (Kutilek, 2004). This may explain the high variation of  $K_s$  and  $\alpha$  within certain soil texture class (Fig. 3). From a practical point of view, the Wu1 might be better because the Wu1 method estimated valid (normally positive) values of  $K_s$  and  $\alpha$  for more cases than the BEST methods (Fig. 2). However, further research is required to investigate why the BEST\_slope and BEST\_intercept methods sometimes provide significantly different estimates of  $K_s$  and  $\alpha$  as shown in this study (Fig. 3).

## 5.3. Implications of this study

In this study, we assessed BEST and Wu methods based on two aspects. The first one is to quantitatively assess their performance in fitting infiltration data using  $Er$  as an indicator of accuracy; the second is qualitatively to assess whether they (BEST and Wu) produced too many “unreasonable” estimates (positive but extremely low values). This is similar to the assessment methods used by Lassabatere et al. (2006) and Yilmaz et al. (2010). The aim of this study is not to use one method (model) to defeat another, but to get some complemented knowledge from different methods so that we can adapt the existing or develop a new method based on the existing knowledge.

Some problems (null and positive but extremely low values) occurred for the Irish soils (under wet conditions) in this study and were also found by other studies (Lassabatere et al., 2010; Yilmaz et al., 2010), and therefore the results of this study is not Irish-soil-specific and not uncommon in the world. This study calls for more research towards advancing these methods for wider conditions (e.g., sandy soils, basic oxygen furnace slag, soils under wet conditions). In addition, not only the estimating methods themselves should be improved but also the field operation should also be given further attention. This may include, for example, minimizing the disturbance when levelling the soil surface, preventing any leaking from the perimeter of the infiltration ring, softly but quickly pouring water into the ring, and carefully sampling the saturated part of soils for saturated soil water content measurement.

## 6. Conclusions

This study evaluated the performance of the BEST (BEST\_slope and BEST\_intercept) and Wu (Wu1 and Wu2) methods in analyzing single-ring infiltrometer data to estimate the soil hydraulic properties  $K_s$  and  $\alpha$  for four soil texture classes. The estimates of these methods appear reasonable, but some problems remain (e.g., poor performance in fitting cumulative infiltration curve for wet conditions, the occurrence of null and extremely low

estimates). Therefore, further research is still required to improve these inversion estimation algorithms themselves as well as the field operations for infiltration experiments.

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