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Multifractal Analysis of Hourly Precipitation

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Abstract. The multifractal nature of the rainfall field is analysed using the methodology of singular measures. The analysis is applied to a long time series (54 years) of hourly rainfall intensities recorded at Valentia on the South-West Coast of Ireland. The empirical probability distribution function suggests a hyperbolic intermittency with the divergence of the statistical moments being higher than the second order. The latter is in agreement with findings of other authors for similar climatic regions (e.g. Sweden). The Fourier transform statistics of the data are used to obtain the scaling range in which the data obey a power law with a coefficient of ~0.5. The scale invariance as identified by the spectral power law, ranges from 2 hours to about 24 hours. This is a narrower range than has been found for similar studies using continental sites where the range was found to be from 2 hours to about 3 days. Studies of Valentia rainfall using conventional statistics suggest that two distinct periods, (1940-1975 and 1976-1993) are clearly present. The second period is characterised by a greater annual rainfall depth than the first, and the increased depth was found to be concentrated primarily in the months of March and October. The intermittency analysis of the rain field of the two periods reveals two different *K(q)*-functions. The curvature of the *K(q)* convexity has been found to be larger for the second period suggesting lower intermittency or more frequent rain events. The intermittency function C_1 for the period 1976-1993 is shown to be quantitatively less than C_1 for the period 1940-1975, for the annual, March and October time series verifying increasing precipitation since 1975.

1. Introduction.

The fundamental source term driving hydrological runoff process and hence river flow fluctuations is the rain field. The scaling properties of this field and many other geophysical fields (e.g. liquid water in clouds) have been the subject of considerable investigation over the past 15 years. At first, several attempts were made to estimate the fractal dimension (presumed monofractality) of rainfall (Lovejoy and Mandelbrot, 1985). However, during the 1980's it became clear that the appropriate framework for hydrological and precipitation analysis should be multifractal (Schertzer and Lovejoy, 1985, Rodriguez-Iturbe and Rinaldo, 1997).

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Precipitation fluctuations in time (and space) are due to a wide range of physical processes which range from climate dynamics to water droplet formation. The dynamics of rainfall and river runoff phenomena may be characterised by a wide range of scales which exhibit scale invariance, i.e., fluctuations at small scales are related to fluctuations at larger scales by the same scaling law. The scaling behaviour within a frequency range may be useful to characterise the time scale of the rainfall and runoff phenomena and may be examined using spectral analysis. Information on spectral peaks is trivial and does not provide as much detail as does the characterisation of geophysical phenomena by spectral scaling laws over a range of scales.

In the past two years there is a growing interest in rainfall parameterisation through multifractal approaches (Tessier et al., 1996; Svensson et al., 1996; Olsson and Niemczynowicz, 1996; Harris et al., 1996; Veneziano et al., 1996; Schmitt et al., 1998). In this paper, scaling methods are applied to a precipitation time series to identify its scaling behaviour. We apply a multifractal data analysis technique, recently proposed by Davis et al., (1994) to study complex non-linear geophysical processes observed over a large range of space and time scales. The technique is aimed at investigating non-stationarity and intermittency as two complementary features of the geophysical data.

In this presentation we examine (i) the scaling behaviour through spectral analysis and (ii) parameterisation of the rainfall based on the study of its main peculiarity, that of intermittency.

2. Rainfall Data

The database comprises 54 years of hourly rainfall from 1940 to 1993 at the Valentia meteorological station on the South-West coast of Ireland (51°52'N, 10°23'W and 9m above sea level). The data were collected by the Irish meteorological service who advise that there has been no change in rain gage surroundings over the period of record. The climate on the west of Ireland is temperate maritime moderated by the warm Gulf Stream. The prevailing wind direction is from the South-West, and these winds tend to bring rain from the Atlantic. The annual rainfall is about 1400mms, with rainfall in all months. In summer the monthly amounts are about 75mms while in the winter they are about 150mms.

Figure 1 shows the hourly rainfall for a typical year (1949). In winter the wet hours per month are as high as 80% of the time, while in summer as much as 50%. Rainfall intensities rarely exceed 10mms/hour while winter hourly intensities are typically 2-4mms/hour. From Figure 1, it is clear that rainfall is an all year round phenomena with increased intermittency in the summer months.

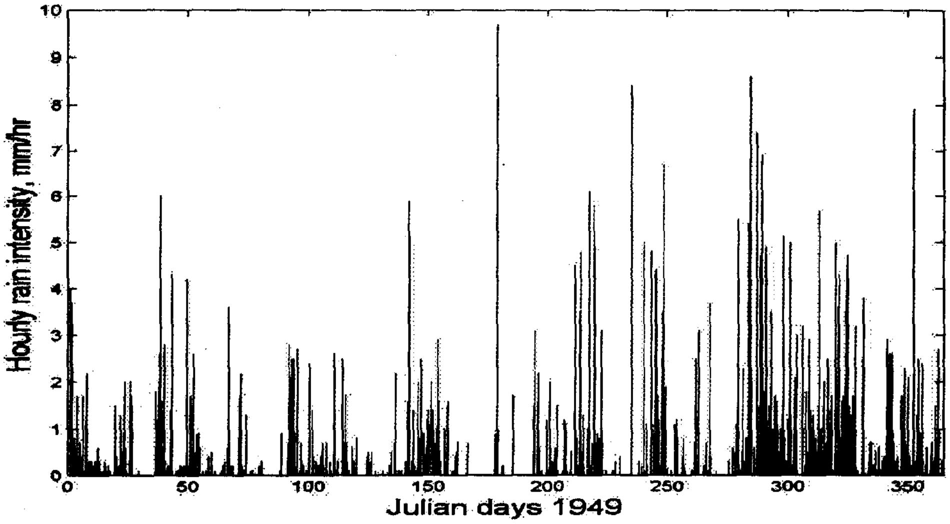

Figure 1. A typical year (1949) of hourly rainfall at Valentia.

3. Methodology.

3.1 Scaling tests.

The hourly rainfall data exhibit fluctuations at a large variety of scales. The simple scaling properties of the rainfall data are examined using two standard techniques: the power spectrum and the empirical probability distribution. The power spectrum is of the form:

$$E(f) \propto f^{-\beta} \qquad (1)$$

and is obtained as a Fourier transform of the data and suggests the range in which scaling exists. The value of the spectral exponent β provides information on the stationarity of the signal. If 0< β <1 the data series is said to be stationary. If β >1 the signal is said to be non-stationary and if β <3 the signal is non-stationary with stationary increments (Monin and Yaglom, 1975). In the latter case, it is essential to conduct the analysis over the small-scale gradient field (Davis et al., 1994, Ivanova and Ackerman, 1998).

The empirical probability distribution function describes the scaling of the intensity fluctuations at a given scale, usually equal to the resolution of the data (Fraedrich and Larnder, 1993). The probability density function *Pr (X > x)* defines the probability of observed intensities exceeding a fixed threshold *x*. If the signal is characterised by hyperbolic intermittency (Lovejoy and Mandelbrot, 1985) the tail of the *pdf* is scaled as

$$\Pr(X > x) \propto x^{-q_D} \qquad (2)$$

Equation (2) defines the critical order value of q_D after which the statistical moments diverge (Schertzer and Lovejoy, 1987). This is interpreted for orders lower than the critical order $q = q_D$ for all values of the time series that contribute to the average moments. For $q > q_D$ only the extreme (maximum) events influence the moments.

3.2 Analysis for Intermittency through singular measures

The singular measure analysis of the small-scale gradient field obtained from the data set, $\varphi(x_i)$, $i = 0,\ldots,\Lambda$, is performed to account for the intermittency. The procedure includes: (i) taking the small-scale differences

$$\Delta\varphi(1;l) = \varphi(x_{i+1}) - \varphi(x_i), \quad i = 0,\ldots,\Lambda-1 \qquad (3)$$

(ii) applying absolute values; and (iii) with some optional normalising we then end up with

$$\varepsilon(1;l) = \frac{|\Delta\varphi(1;l)|}{\langle|\Delta\varphi(1;l)|\rangle}, \quad x = 0,\ldots,\Lambda-1 \qquad (4)$$

where $$\langle|\Delta\varphi(1;l)|\rangle = \frac{1}{\Lambda}\sum_{l=0}^{\Lambda-1}|\Delta\varphi(1;l)| \qquad (5)$$

The procedure continues with coarse screening of data by performing spatial averages. We derive a series of ever more coarse screened values and, therefore, ever smaller fields $\varepsilon(r;l)$ averaged over r sized boxes for $r = 1,2,\ldots,\Lambda = 2^m$. (The assumption of the size being an integer power of 2 is not essential for the outcome.)

$$\varepsilon(r;l) = \frac{1}{r}\sum_{l'=l}^{l+r-1}\varepsilon(1;l'), \quad x = 0,\ldots,\Lambda-r \qquad (6)$$

After each step in r an ensemble average is taken and the result is denoted by $<\varepsilon(r;l)>$. Then the scaling properties of this new quantity are studied with respect to r. Furthermore, not only the first order statistical moments but arbitrary qth order statistics $<\varepsilon(r;l)^q>$ can be derived. Their power law behaviour with respect to r is also sought to behave like

$$\langle\varepsilon(r;l)^q\rangle \propto (r)^{-K(q)}, q \geq 0 \qquad (7)$$

The method continues to calculate both a (non-decreasing) function $C(q)=K(q)/(q-1)$ and a (non-increasing) function $D(q)=1-C(q)$. The latter is the hierarchy of the so-called generalised dimensions studied by Hentschel and Procaccia (1983) and by Grassberger (1983) along the lines of deterministic chaos theory. D_0 is the usual box-counting fractal dimension and has meaning only for self-similar fractals. D_1 is the information dimension stemming from the definition of entropy in the information theory. D_2 is the correlation dimension and measures the scaling of the correlation function (Hentschel and Procaccia 1983, Grassberger, 1983, McCauley, 1990). If $D(q)$ varies with q, a multifractal behaviour exists. We restrict our considerations to the information dimension D_1.

L'Hospital's rule in the limit of $q \rightarrow 1$ provides a straightforward measure of the in-homogeneity of the field that defines the intermittency parameter C_1. The sparseness of the signal is characterised by C_1. A large C_1 value describes a high level of intermittency and spikeness and therefore small values of information estimated through D_1. However, C_1 is not fully sufficient to describe the multifractal properties of a signal. This is obtained by the entire spectrum of $K(q)$ values.

4. Data Analysis and Results.

4.1 Simple scaling and order of divergence of moments.

The scaling properties of the rainfall data are first examined. Previous studies by Kiely et al. (1998) of the same data using conventional statistics identified two periods (1940-1975 and 1976-1993) with distinctly different summary statistics. The post 1975 period has an annual rainfall 10% greater than that of the pre-1975 period. Furthermore this increase is mostly contained in the months of March and October. The increase is due to increasing frequency of westerly rain bearing winds as seen in the changes to the North Atlantic Oscillation (Kiely, 1999). We compute the power spectrum for the full period 1940-1993 and for the two sub periods (pre and post 1975). Figure 2 shows the power spectrum for the full period.

From the spectral plot we see clear spectral peaks at one year and at six months indicative of intrinsic periodicities of the annual and half annual cycle.

There are several distinct scaling regions in the spectral plot.

(a). *1 month to > 1 year*. This region exhibits a flat or spectral plateau with a spectral exponent of $\beta=0$. This region governs inter-seasonal and intra-seasonal variability.

(b). *1 day to 1 month*. It may be considered as a transition region between region (a) and (c) with a scaling exponent of $\beta=0.12 \pm 0.03$

(c). *2 hours to 1 day*. This region governs frontal weather systems. The main characteristic of this region of the spectra is in the scaling of this high frequency end with an exponent of $\beta = 0.52\pm 0.01$. This value of the exponent indicates stationarity of the rainfall events. The range in which the scaling properties holds, spans from twice the 1 hour discretization interval to ~1day. Fraedrich and Larnder (1993) reported an upper scaling limit for frontal systems of approximately 3 days from a rainfall series from various European stations. Scaling up to three days is reported also for high resolution (~8min) rainfall data from Sweden (Olsson, 1995). We interpret our findings of an upper scaling region of ~1day to be specific to the local maritime climate characteristics at Valentia. The west of Ireland is a first landfall point for Atlantic frontal systems from the south-west.

The data used is hourly observations so the spectra do not enter the very high frequency range (less than 1 hour). Downscaling from hourly to 5-minute storms is an exercise yet to be undertaken.

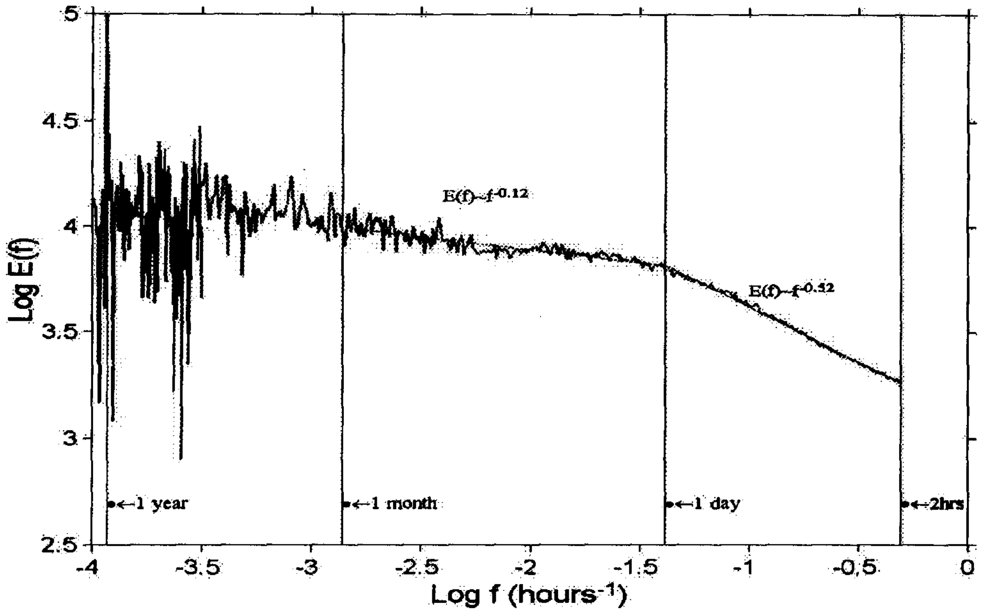

Figure 2. Power spectrum of the hourly rainfall. The power is averaged over logarithmically spaced frequency intervals. The straight lines represent the linear fits of $E(f) \propto f^{-\beta}$ with the exponent β=*0.5* from 2hrs to 1 day and an exponent of 0.12 from 1 day to 1 month. The exponents are the same for the full period (1940-1993) and for the two sub periods (1940-1975 and 1976-1993

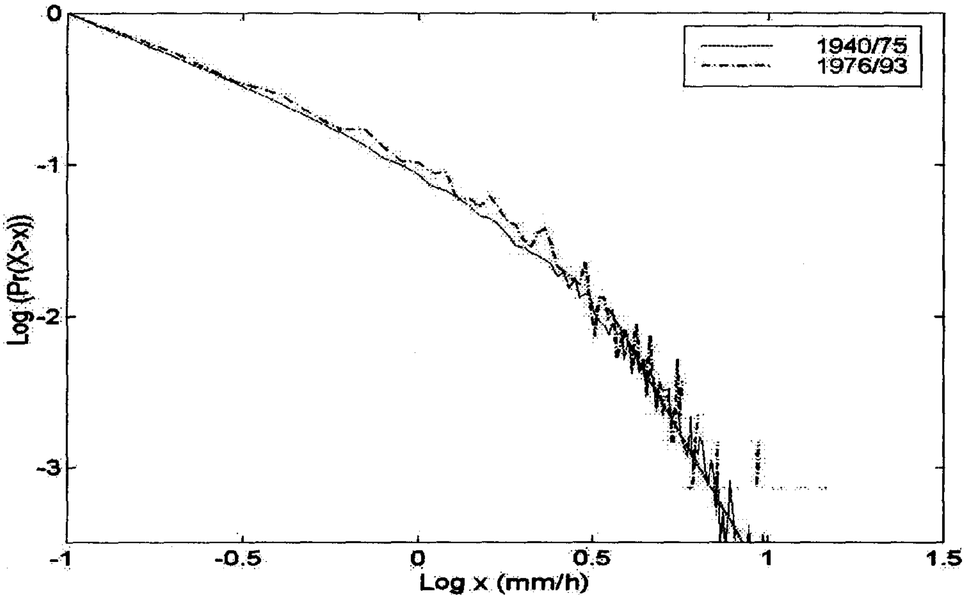

Figure 3. Empirical probability density function (normalised) *Pr(X>x)* for the two periods. The linear fit of the tail of $\Pr(X > x) \propto x^{-q_D}$ defines the largest order $q_D=4$ after which the moments will diverge.

In analysing the power spectra and the empirical probability density $Pr(X>x)$ we consider the entire length (including hours of zero rain) of the rainfall series for each period (pre and post 1975). In Figure 3 the (normalised) probability density functions for the two periods are drawn. The linear fit of the tail of the distributions represented by (the same) straight lines for both cases of interest defines the largest order after which the moments will diverge, that is $q_D=4$. Therefore, the multifractal analysis should be performed for orders of moments for $q \leq 4$.

4.2 Multifractal properties.

The multiscaling properties of the rainfall field were examined using the $K(q)$ spectrum for positive moment orders q smaller and equal to q_D The results for $q<0$ lead to large uncertainties so that the numerical analysis is somewhat less relevant in this part of the spectrum. The same implies to the $H(q)$ spectrum which characterises the structure functions (Vandewalle and Ausloos, 1998).

For each year the $<\varepsilon(r;l)^q>$ measure is calculated for fixed value of $q \in [0.1, 4]$. Then for each q the ensemble average is calculated over the first (1940-1975) period and the second (1976-1993) period. The scaling properties of the averaged singular measures are tested using Equation (7). By fitting with a line of $<\varepsilon(r;l)^q>$ vs. r in a log-log plot we obtain the values of the $K(q)$ function for fixed q. The $K(q)$ curve is presented in Figure 4 for the two periods. The error bars indicate the linear fitting precision.

It should be noted that the increasing behaviour of $K(q)$ is unlike the minimum containing curve for $0<q<1$ obtained when analysing other atmospheric quantities such as liquid water in clouds (Davis et al., 1994; Ivanova and Ackerman, 1999). The difference is due to the fact that we analyse the raw rainfall data. If we modified the data by extracting the dry hours, i.e. zeros of the time series, then the $K(q)$ function would possess a minimum for q in the interval $0<q<1$ (Schmitt, 1998). Our result is in agreement with Olsson's (1995) findings for $K(q)$ rainfall scaling. The above discrepancy does not alter the overall non-linear behaviour of the $K(q)$ spectrum. It is the convexity of this characteristic exponent that determines the multifractality of the rainfall field

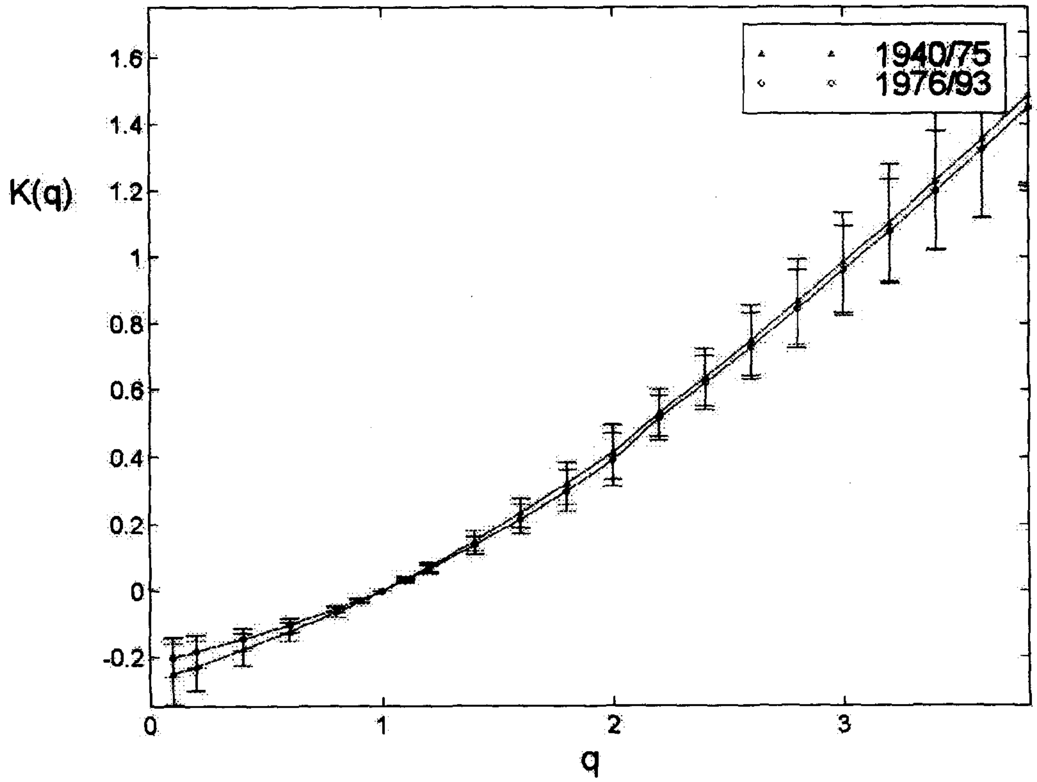

Figure 4. The characteristic intermittency exponent $K(q)$ as defined by $\left\langle \varepsilon(r;l)^q \right\rangle \propto r^{-K(q)}, \quad q \geq 0$. The empty circles apply to the period 1976-1993 and the open triangles are for the period 1940-1975.

4.3 Intermittency (C_1) results.

We examine the two periods for intermittency C_1. In Table 1 the C_1 parameter is shown for the annual series. It is seen that the intermittency is higher in the early period (1940-1975). This suggests increased rainfall in the second period. When we compute C_1 for the March and October time series we find similar results, i.e. increasing rain in the post 1975 period. However when we compute the intermittency for the month of May, there is little difference in C_1 for the pre and post 1975 periods.

5. Discussions and Conclusions.

We have analysed 54 years of hourly rainfall data at a coastal site on the south-west of Ireland using multifractal techniques. Analyses included investigations of the power spectra for scaling behaviour and investigations of intermittency using the concept of singular measures for quantifying differences between two periods. Further work is to do spectral analysis of two years of 5 minute data with the view to scaling in the storm or sub 1 hour range.

Table 1. INTERMITTENCY (C1) parameters for the period I (1940-1974) and period II (1975-1993) for the Annual, March, May and October, hourly rainfall time series at Valentia, Ireland.

Season	C1 Period 1 1940-1974	C1 Period 2 1975-1993	ΔC1 Period 1- Period 2
Annual	0.344	0.303	0.041
March	0.372	0.338	0.034
May	0.403	0.390	0.013
October	0.373	0.353	0.020

The analysis showed that scaling of the power spectra at the high frequency end has an exponent β of 0.52 corresponding with a range of 2 hours to 24 hours. This temporal range and exponent holds for the full period 1940-1993 and for the two distinct periods, 1940-19975 and 1976-1993. This upper end of the scaling range of 1 day is lower than that the 3 days found by other authors.

Analysis of K(q) scaling verify that this rainfall data set is multifractal.

Analysis of intermittency (C_1) for the two periods (pre and post 1975) indicates a substantial reduction in intermittency from pre to post 1975 periods. This is also highlighted in the specific months of March and October. The results from the multifractal analysis correlates with results found earlier using conventional statistics that Valentia has experienced increased rainfall since 1975.

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