



PHYSICAL AND MATHEMATICAL MODELLING OF ANAEROBIC DIGESTION OF ORGANIC WASTES

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Abstract—Anaerobic digestion of the organic food fraction of municipal solid waste (OFMSW), on its own or co-digested with primary sewage sludge (PSS), produces high quality biogas, suitable as renewable energy. We report the results from one such bench scale laboratory experiment, on the co-digestion of OFMSW and PSS. The experiment used a continuously stirred tank reactor and operated at 36°C for 115 days. Prior to the experiments, activity tests verified that the inoculum sludges were suitable for the biodegradation of the volatile fatty acid substrate and so producing biogas. The experimental data were used to develop and validate a two-stage mathematical model of acidogenesis and methanogenesis. In simulating the behavior of the anaerobic digestion process, including ammonia inhibition, the mathematical model successfully predicts the performance of methane production. Simulations of the pH and ammonia in the MSW anaerobic reactor were also satisfactory. Sensitivity analysis on the 18 model parameters indicated that eight of these parameters were in the most sensitive and highly sensitive range, while the remainder were in the moderate to least sensitive range. © 1997 Elsevier Science Ltd. All rights reserved

Key words—anaerobic digestion, municipal solid waste (MSW), primary sewage sludge (PSS), pH, ammonia, alkalinity, biogas, methane, carbon dioxide, mathematical modelling

INTRODUCTION

The process of anaerobic digestion of the organic fraction of municipal solid waste (OFMSW), on its own or co-digested with other organic sludges (e.g. primary sewage sludge, PSS), has the potential to contribute significantly to the renewable energy budget and also to the reduction of landfill or other undesirable waste disposal routes. The use of OFMSW is proving itself to be technologically feasible in reducing the volumes of waste generated by producing biogas (Jewell, 1979; Kayhanian and Tchobanoglous, 1990; Vallani, 1992; Cout *et al.*, 1994). The co-digestion of OFMSW with other fractions, including primary sewage sludge, industrial organic sludges (e.g. from the food processing industry) and agricultural slurries has also been investigated, and is operational at full scale in several centralized co-digestion plants in Denmark (Danish Energy Agency, 1994, 1995). The Danish Energy Agency (1994) reports on 18 plants in operation in the European Union with at least some fraction of the feedstock being OFMSW. Many of these are high solids processes and most are of proprietary patented design, with 12 operating at mesophilic temperatures. There are according to the Danish Energy Agency (1994), approximately a further 26 plants under

construction or being planned world wide, with feedstock mixtures of OFMSW and other organic wastes. The sizes of the above mentioned plants vary from a low of 500 tonnes/year to a high of 95,000 tonnes/year. While these figures might indicate a technology that is well proven, the reality is that there are many operational problems, not only to do with feedstock consistency but also with operational temperature, ammonia inhibition, excess fatty acids etc.

Co-digestion of industrial wastes and agricultural wastes has also been verified by Sugrue *et al.* (1992). An evaluation of the biogas potential from co-digestion of OFMSW, agricultural slurries, municipal sludge and industrial sludges (using four equal fractions) was made by Kiely *et al.* (1995). This study identified that the biogas produced could generate as much as 872 GWh of electricity or 6% of the country's electrical energy requirement. Biogas production has been shown to be about 2 m³/m³ of reactor, for digesters containing low solids mixtures (<5% dry solids) at mesophilic temperatures (Kiely *et al.*, 1994). In contrast, biogas output up to 7 m³/m³ of reactor for high solids mixtures (>20% dry solids) at thermophilic temperatures has been reported by Kayhanian *et al.* (1991) and Norgaard (1995). One of the major problems with the operational stability of anaerobic digesters is inhibition of microbial growth by ammonia (Hashimoto, 1986). Additionally from an engineering standpoint, high solids technology is

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still complicated and costly (Norgaard, 1995). With the above background it was decided to carry out a small scale laboratory anaerobic co-digestion study with OFMSW being the principal feedstock. The objectives of the study were two-fold:

- To determine the feasibility of anaerobic digestion (in a controlled environment) of a mixture of the source separated food fraction of household waste and primary sewage sludge and to quantify the biogas production, and so provide a data base for analysis and mathematical model development.
- To develop a mathematical model for the anaerobic digestion process, and to simulate methane production, using the experimental data for the calibration of the process parameters.

EXPERIMENTAL PROCEDURES

The continuously stirred tank reactor (CSTR) was operated as a bench scale experiment at University College Cork over a period of 115 days. The reactor was 21 in volume and operated at a constant temperature of 36°C in a water bath, with daily feed. The gas collection system consisted of an inverted beaker over a solution of 20% NaCl and 0.5% citric acid. The initial inoculum for the reactor came from two full scale operational anaerobic digesters. The reactor was seeded with 1.5 l of the mixture and allowed to acclimatize for 13 days. The reactor was fed with pig slurry for the next 21 days. The "setting-up" period of 34 days produced a digester with a working inoculum which had a pH of 7.75, an alkalinity of 5.5 g/l and an ammonia level of 1.5 g/l. These are the starting parameters of the experiment as identified for day 1.

The experiment continued for a further 115 days. The substrate from day 1 to day 33 was pig slurry. This had a COD of 38.3 g/l and an influent total solids of 4.2%. The pig slurry was high in alkalinity, ammonia and VFAs. The substrate from day 33 to day 40 was raw primary sewage sludge (only) with a COD of 36.0 g/l and a total solids of 2.1%. The substrate from day 40 to day 115 was a consistent uniform mixture of the organic food fraction of MSW and primary sewage sludge.

The MSW fraction was prepared from a collection of a representative number of households who, over a 12 week period, agreed to source separate their waste into two fractions, namely the food component and all others. Only the food fraction was used in this experiment. The substrate mixture was prepared by adding 125 g of the homogenized food fraction to 650 ml of the primary sewage sludge. This

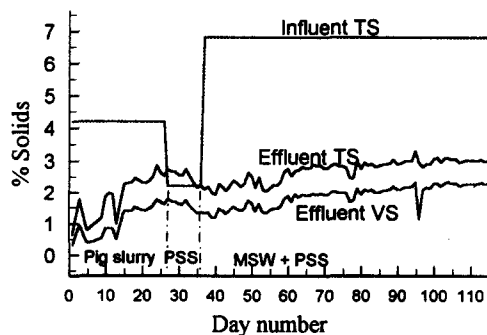


Fig. 1. Influent TS, effluent TS and effluent TVS.

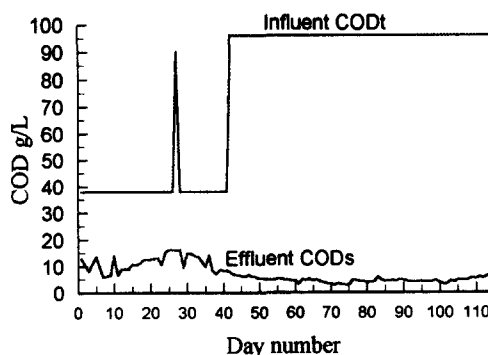


Fig. 2. Influent CODt and effluent CODs.

gave an aqueous solution with a total solids of 6.9%, a COD (total COD) of 96.0 g/l and a pH of 5.4. It was low in alkalinity (average 1.25 g/l) and low in ammonia (average 0.25 g/l). A sufficient feedstock was made up initially to last for the full duration of the experiment. A survey of the households from where the MSW was collected showed that the organic fraction was 37% of the total collected MSW (Mortensen *et al.*, 1989). This is typical for MSW in Europe. However the comparable figure in the United States is about 10% (Kayhanian and Tchobanoglous, 1992), where much of the food waste is deposited in kitchen sink grinders.

EXPERIMENTAL RESULTS

The experimental results are presented in Figs 1–7. Figure 1 shows the influent total solids (TS), the effluent TS and total volatile solids (TVS). The influent TS rises from about 4.25% for the first 30 days to about 7% from day 40 onwards (due to the OFMSW/PSS mixture). The effluent total solids is relatively constant at about 3% after 25 days. The effluent TVS varies from about 50% to 75% of the TS. It should be noted that the "solids" parameter was "total" and not "suspended solids". A stoichiometric analysis of the substrate and stable end products showed that on average about 75% of the VS were degraded, 11% not degraded and the remaining 14% converted to cell tissue. This computes to a biomass concentration of 3.2 g/l.

Figure 2 shows the influent CODt (total) and the effluent CODs (soluble). This parameterization was used, because in real situations the digester is usually

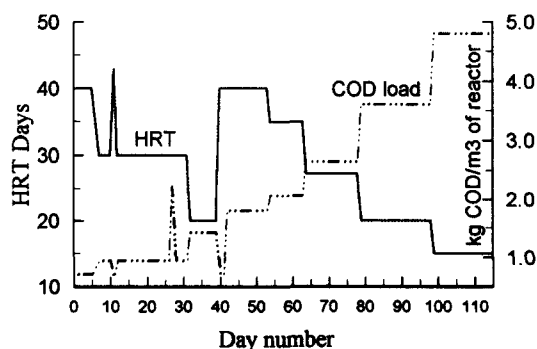


Fig. 3. HRT and COD loading.

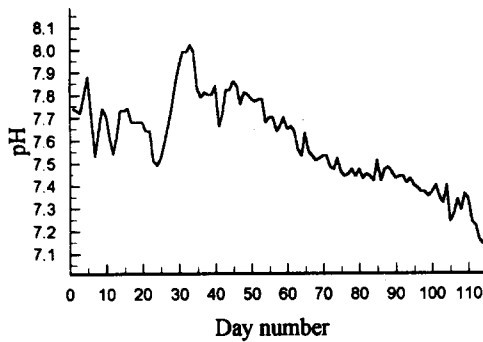


Fig. 4. pH profile.

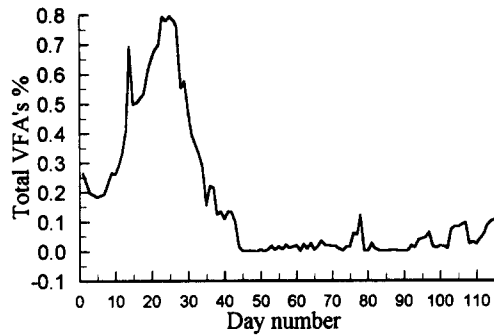


Fig. 6. Total VFAs.

followed by a clarifier and the activated sludge (AS) process for further treatment and the data needed for AS design are preferable as CODs. The influent CODt for the first 40 days was essentially 38 g/l and 96 g/l thereafter. The effluent CODs varied from 5 to 15 g/l for the first 40 days and thereafter stabilized at 5 g/l. A series of discrete total COD tests on the effluent indicated values about 20% higher than that of the soluble COD. The total COD of influent of 96 g/l was reduced to 6 g/l from day 40 onwards, a reduction of 94%.

The total COD load represented as kg/m^3 of reactor per day is shown in Fig. 3. This rises from a low of $1 \text{ kg}/\text{m}^3$ at the beginning of the experiment to a high of $4.8 \text{ kg}/\text{m}^3$ at the end. This emphasizes that the reactor adjusted well to the increased COD load. This compares favorably with data in the literature. Six *et al.* (1994) quote a loading of $12 \text{ kg}/\text{m}^3$ for a high solids MSW thermophilic reactor. The hydraulic retention times are shown in Fig. 3. Hydraulic retention times decreased from a maximum of 40 days to a desirable 15 days for the latter part of the experiment. Figure 3 verifies that the reactor continuously adjusted to the increased hydraulic and COD loading over the course of the experiment.

Figure 4 shows the pH of the reactor contents (effluent). Up to day 40, the pH varied from 7.5 to 8.0 and thereafter (with the OFMSW/PSS mixture) the pH fell gradually to a low of 7.1 on day 115. Figure 5 shows the change in ammonia and alkalinity over

time. For the first 33 days the alkalinity rose from a low of 5.5 g/l to a high of 13.5 g/l. During the same initial period the ammonia increased from 1.5 g/l to 3.75 g/l. There was a rapid fall in alkalinity and ammonia during the 7 day feedstock of primary sewage sludge (PSS). From day 40 onwards (feedstock of OFMSW/PSS), the alkalinity fell from 10 to 3.5 g/l, while the ammonia fell from 2.5 to 0.5 g/l. The ammonia levels (below 2.5 g/l) were low enough so as to avoid inhibition problems.

Figure 6 shows the variation of total VFAs over time. The total VFAs increased from day 5 onwards and then peaked at about day 25 at a magnitude of 0.8%. There was a rapid decrease in total VFAs after day 25. As PSS was not fed to the reactor until day 33, initially it was expected that the VFAs would increase up to day 33. However, from day 25 onwards, the methanogens increased (as shown by the increasing biogas production; see Fig. 7), therefore consuming the VFAs and so lowering the total VFAs to nearly zero from day 40 onwards.

Figure 7 shows the biogas and methane output in m^3/m^3 of reactor per day. There is practically no gas output up to day 20. This is followed by a dramatic increase until day 25 when the biogas is $0.75 \text{ m}^3/\text{m}^3$. A fall to $0.5 \text{ m}^3/\text{m}^3$ was experienced from day 33 to day 35 due to a change in feedstock. From day 35 onwards there was a steady increase up to a peak of $2.38 \text{ m}^3/\text{m}^3$ at day 106. There was a small decrease to $2.0 \text{ m}^3/\text{m}^3$ thereafter. A few minor decreases occurred

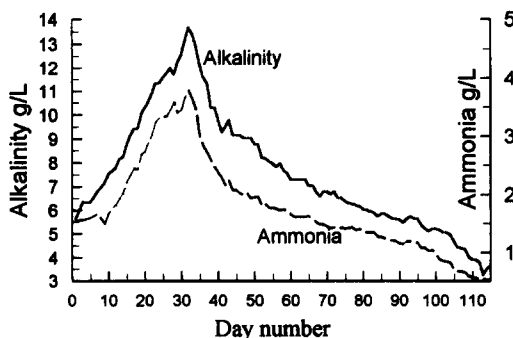


Fig. 5. Alkalinity and ammonia.

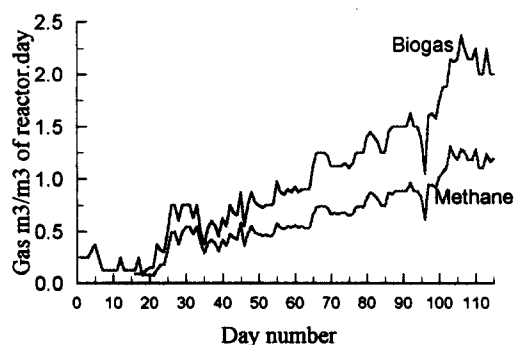


Fig. 7. Biogas and methane production.

between day 40 and day 115, but these were not so significant to prevent the upward trend in biogas production. The methane percentage varied from 50 to 60% of biogas. At the peak biogas production, the methane content was 54% of biogas.

The results shown in Figs 1–7 verify that the reactor was robust during its 115 days of operation, showing that a mixture of OFMSW and PSS can be anaerobically digested at mesophilic temperatures, producing respectable amounts of biogas.

MATHEMATICAL MODEL

Model development

The mathematical formulation is based on Hill and Bart (1977), Havlik *et al.* (1986) and Moletta *et al.* (1986). The model considers the production of methane as being two-stage; that of hydrolysis/acidogenesis producing acetate and that of acetoclastic methanogenesis producing methane. The variation of acidogenic biomass with time is:

$$\frac{dX_a}{dt} = \mu_a X_a - K_{d_a} X_a, \quad (1)$$

where X_a = acidogenic biomass (g/l); K_{d_a} = death rate of acidogenic biomass; and μ_a = the specific growth rate of acidogenic biomass (day^{-1});

$$\mu_a = \frac{\mu_{a_{\max}}}{1 + \frac{K_{x_a}}{S} + \frac{A_h}{K_{i_a}}}, \quad (2)$$

where $\mu_{a_{\max}}$ = the maximum specific growth rates; K_{x_a} = the saturation constant for the acidogenic bacteria growth (g/l); S = glucose equivalent concentration of the substrate (g/l); A_h = unionized acetic acid concentration (g/l); and K_{i_a} = the inhibition constant of acidogenic bacteria growth (g/l). The rate of change in the glucose equivalent concentration of the substrate is:

$$\frac{dS}{dt} = D(S_{\text{inf}} - S) - \frac{\mu_a X_a}{T_a} + \frac{\mu_a X_a}{Y_{\text{so}}}, \quad (3)$$

where D = dilution rate (day^{-1}); S_{inf} = influent glucose equivalent concentration of the substrate (g/l); Y_a = yield coefficient for acid-formers (g/g); and Y_{so} = soluble organic yield coefficient (g/g). On the RHS of (3), the second term represents the utilization of glucose by acid-formers in their metabolism and the third term represents the yield of glucose from volatile matter. The unionized acid concentration is:

$$A_h = \frac{AH^+}{K_c}, \quad (4)$$

where A = total acetic acid concentration (g/l); K_c = dissociation constant for acetic acid at 35°C, which is equal to 1.728×10^{-5} ; and H^+ =

hydrogen ion concentration. The rate of change of total acetic acid concentration is:

$$\frac{dA}{dt} = D(A_{\text{inf}} - A) + \frac{\mu_a X_a}{Y_{va}} - \frac{\mu_m X_m}{Y_m}, \quad (5)$$

where A_{inf} = influent acetic acid (g/l); Y_{va} = yield of acetic acid (g/g); μ_m = specific growth rate of methanogenic biomass (day^{-1}); X_m = methanogenic biomass (g/l); and Y_m = yield coefficient for methane formers (g/g). The variation of methanogenic biomass with time is:

$$\frac{dX_m}{dt} = \mu_m X_m - K_{d_m} X_m, \quad (6)$$

where K_{d_m} = the death rate for methanogenic biomass. The specific growth rate for methanogenic biomass is:

$$\mu_m = \frac{\mu_{m_{\max}}}{1 + \frac{K_{x_m}}{A_h} + \frac{A_h}{K_{i_m}} + \frac{NH_3 M_{NH_3}}{K_{i_m}}}, \quad (7)$$

where $\mu_{m_{\max}}$ = the maximum specific growth rate for the methanogenic biomass (day^{-1}); K_{x_m} = the saturation constant for the methanogenic bacteria growth (g/l); NH_3 = concentration of the un-ionized ammonia (mol/l); M_{NH_3} = mole weight of NH_3 (g/mol); and K_{i_m} = the inhibition coefficient of ammonia (g/l). The rate of methane production as expressed by Moletta *et al.* (1986) is:

$$\frac{dCH_4}{dt} = V_{m_{\max}} X_m \left(\frac{A_h}{A_h + K_m} \right) \left(\frac{K_{i_m}}{K_{i_m} + A_h} \right), \quad (8)$$

where CH_4 = methane production (g/l); $V_{m_{\max}}$ = maximal production rate (in g) per g of methanogenic bacteria per day (g/g day); K_m = saturation constant of methane production (g/l); and K_{i_m} = the inhibition constant of acetic acid on methane production (g/l).

pH and ammonia

pH is computed using a mass balance of the CO_2 system as follows:

$$\begin{aligned} \frac{dCO_2}{dt} = & D[CO_{2_{in}} - CO_2 + HCO_{3_{in}} - HCO_3] \\ & + R_M + R_{AC} + R_{AF} - R_Z - R_{NH_4} + R_T, \quad (9) \end{aligned}$$

where $CO_{2_{in}}$ = the influent dissolved CO_2 concentration (mol/l); CO_2 = the effluent dissolved CO_2 concentration (mol/l); $HCO_{3_{in}}$ = influent bicarbonate concentration (mol/l); and HCO_3 = effluent bicarbonate concentration (mol/l).

(A) R_M , the rate of CO_2 formation by methane production (mol/l d), is:

$$R_M = \frac{\mu_m X_m Y_{CO_{2_m}}}{M_x}, \quad (10)$$

where $Y_{CO_{2_m}}$ = the CO_2 yield coefficient for methane formers (mol/mol); and M_x = assumed mole weight of organisms (113 g/mol).

(B) R_{AC} , the rate of CO_2 production from HCO_3^- by acid formation (mol/l d), is:

$$R_{AC} = \frac{dA}{dt} \left(\frac{1}{M_a} \right), \quad (11)$$

where M_a = the mole fraction of acetic acid (g/mol).

(C) R_{AF} , the rate of CO_2 formation by acid formation (mol/l d), is:

$$R_{AF} = \frac{\mu_a X_a Y_{\text{CO}_2 a}}{M_x}, \quad (12)$$

where $Y_{\text{CO}_2 a}$ = the CO_2 yield coefficient for acid formers (g/g).

(D) R_Z , the rate of CO_2 consumption in the HCO_3^- forming reaction due to cation (other than NH_4^+ and H^+) release from the primary substrate (mol/l d), is:

$$R_Z = \frac{dZ}{dt}, \quad (13)$$

where Z represents the effluent concentration of cations (other than NH_4^+ and H^+) (in eq/l day), and the mass balance for Z is expressed as:

$$\frac{dZ}{dt} = D(Z_{\text{in}} - Z) + \mu_a X_a Y_{\text{cat}}, \quad (14)$$

where Z_{in} = the influent concentration of cations (other than NH_4^+ and H^+) (in, for e.g., per l); Y_{cat} = the yield coefficient of cations other than NH_4^+ and H^+ (e.g. per l).

(E) R_{NH_4} , the rate of CO_2 consumption in the HCO_3^- forming reaction due to NH_4^+ release of the primary substrate (mol/l d), is:

$$R_{\text{NH}_4} = \frac{d\text{NH}_4}{dt} \left(\frac{1}{M_{\text{NH}_4}} \right), \quad (15)$$

where M_{NH_4} is the mole weight of NH_4 (g/mol); NH_4 is the effluent concentration of ammonium (g/l), and the mass balance for NH_4 is expressed as:

$$\frac{d\text{NH}_4}{dt} = D(\text{NH}_{4\text{in}} - \text{NH}_4) + \mu_a X_a Y_{\text{NH}_4} + R_{\text{NH}_3} M_{\text{NH}_4} \quad (16)$$

$$R_{\text{NH}_3} = K_{\text{la}}(K_{\text{H}_{\text{NH}_3}} P_{\text{NH}_3} - \text{NH}_3), \quad (17)$$

where $\text{NH}_{4\text{in}}$ = the influent NH_4 concentration (g/l); Y_{NH_4} = the yield coefficient of ammonium from the raw waste (g/g); K_{la} = the gas transfer coefficient (day^{-1}); $K_{\text{H}_{\text{NH}_3}}$ = the Henry's law gas constant for ammonium (5.33×10^{-6} mol/mmHg l at 35°C); and

P_{NH_3} = the partial pressure of NH_3 in the gaseous phase (mmHg), and the rate of change is expressed as:

$$\frac{dP_{\text{NH}_3}}{dt} = - \frac{T_p S_v V_{\text{rec}} R_{\text{NH}_3}}{V_{\text{gsv}}} - \frac{P_{\text{NH}_3} Q}{V_{\text{gsv}}}, \quad (18)$$

where T_p = the total pressure of CO_2 , CH_4 and NH_3 in the gas storage unit (assumed to be at 730 mmHg); S_v = the standard volume (25.4 l/mol at 35°C); V_{rec} = reactor volume (l); and Q = the total gas flow (l/day), expressed as:

$$Q = Q_{\text{NH}_3} + Q_{\text{CH}_4} + Q_{\text{CO}_2} \quad (19)$$

$$Q_{\text{NH}_3} = - S_v V_{\text{rec}} R_{\text{NH}_3} \quad (20)$$

$$Q_{\text{CH}_4} = (S_v V_{\text{rec}})(\mu_m X_m Y_{\text{CH}_4}) \left(\frac{1}{M_{\text{CH}_4}} \right) \quad (21)$$

$$Q_{\text{CO}_2} = - S_v V_{\text{rec}} R_T, \quad (22)$$

where Y_{CH_4} = the yield coefficient for methane forming bacteria (mol/mol); and M_{CH_4} = the mole weight of CH_4 (g/mol).

(F) R_T , the rate of CO_2 transfer to the gas phase (mol/l d), is:

$$R_T = K_{\text{la}}(K_{\text{H}_{\text{CO}_2}} P_{\text{CO}_2} - \text{CO}_2), \quad (23)$$

where $K_{\text{H}_{\text{CO}_2}}$ represents the Henry's law gas constant for CO_2 (3.47×10^{-5} mol/mmHg l at 35°C); P_{CO_2} represents the partial pressure of CO_2 in the gaseous phase (mmHg), and the rate of change of P_{CO_2} is expressed as:

$$\frac{dP_{\text{CO}_2}}{dt} = - \frac{T_p S_v V_{\text{rec}} R_T}{V_{\text{gsv}}} - \frac{P_{\text{CO}_2} Q}{V_{\text{gsv}}}. \quad (24)$$

In order to calculate the pH, the HCO_3^- concentration is computed before the carbonate equilibrium is established. This is obtained through the charge balance between the cations and anions within the reactor.

The bicarbonate concentration, hydrogen ion concentration, pH and ammonia concentration are defined in equations (25)–(28):

$$\text{HCO}_3^- = Z^+ + \frac{\text{NH}_4^+}{M_{\text{NH}_4}} - \frac{A_{\text{H}^+}}{M_{\text{O}}} \quad 5.0 < \text{pH} < 8.0 \quad (25)$$

$$\text{H}^+ = \frac{K_{\text{CO}_2} \text{CO}_2}{\text{HCO}_3^-} \quad (26)$$

$$\text{pH} = - \log(\text{H}^+) \quad (27)$$

$$\text{NH}_3 = \frac{\text{NH}_4 K_{\text{NH}_4}}{\text{H}^+ M_{\text{NH}_4}}, \quad (28)$$

Table 1. Initial values

Acetic acid (g/l)	Dissolved CO_2 (mol/l)	Cations (mol/l)	Ammonium (g/l)	PP of CO_2	PP of NH_3	pH
1.0	0.011	0.006	1.5	20.0	20.0	7.7

PP = partial pressure in mmHg.

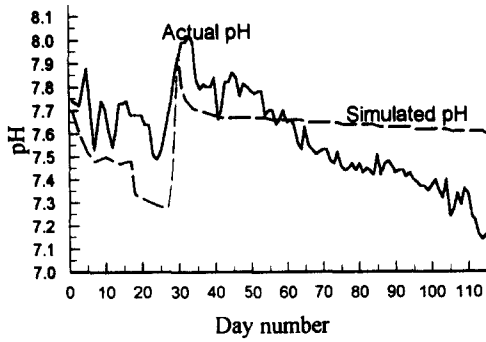


Fig. 8. Actual and model-simulated pH.

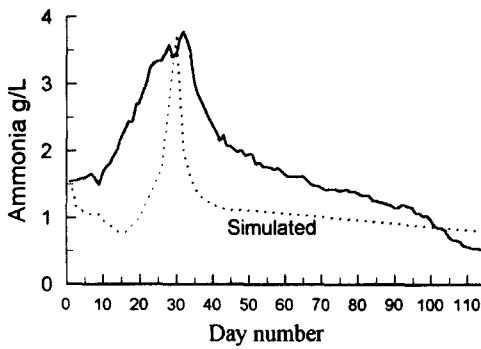


Fig. 9. Actual and model-simulated ammonia.

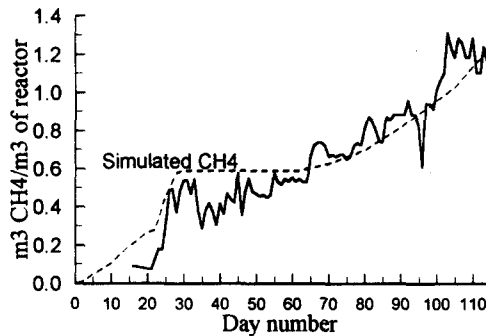


Fig. 10. Actual and model-simulated methane production.

where K_{CO_2} is the CO_2 ionization constant (4.72×10^{-7} at $35^\circ C$) and K_{NH_4} is the NH_4 ionization constant (5.3×10^{-10} at $35^\circ C$).

Model solution

Equations (1)–(28) were solved numerically using the fourth order Runge–Kutta method. The time step chosen was 5.6 min (or 0.0016 days), as the smaller the time step, the closer the approximate solution is to the actual one. Initial values for pH, ammonium and acetic acid were those observed on day 1 of the

Table 3. Model parameter sensitivity

Most sensitive	Y_a	Y_{va}	$\mu_{a,max}$		
Highly sensitive	Y_{so}	Y_m	K_{sm}	$\mu_{m,max}$	K_{dm}
Moderately sensitive	Y_{CO_2m}	Y_{CH_4}	Y_{CO_2a}	K_{da}	K_{iam}
Least sensitive	Y_{NH_4}	Y_{cat}	K_{sa}	K_{sam}	K_{isa}

experiment. The initial values of some variables were assumed within a reasonable range of those quoted in the literature. Further initial values are shown in Table 1.

Application of the model

The coded model was used to simulate observed pH, ammonium (NH_4) and methane (CH_4) data obtained from the experimental reactor. The model requires influent values for substrate and COD loading and its glucose equivalent concentration. Also required are the bicarbonate, cation and ammonium concentrations, which were taken as 0.0033 mol/l, 0.016 mol/l and 1 g/l, respectively. The substrate loading determines the dilution rate as $D = \text{substrate load}/V_{rec}$. The glucose equivalent of COD assumes 192 mg of COD is equal to 1 mmol of glucose (Costello *et al.*, 1991). The model assumes a 25 day lag, as suggested by the observed data, after which methane production is in the growth phase. During this lag time, the release of growth and death in bacterial populations was gradual in the model on the 8th, 16th and 25th days. Full release was allowed by the 25th day. Figures 8, 9 and 10 show the observed and model results for pH, NH_4 and CH_4 , respectively. The overall simulation is satisfactory, since there are 19 model parameters to consider and 13 model variables. The key model parameters which result in a satisfactory simulation of pH, NH_4 and CH_4 were obtained through calibration and are shown in Table 2. A sensitivity analysis of parameters was carried out to determine the most important ones. The results are tabulated in Table 3.

CONCLUSIONS

This study presents the results of a laboratory experiment of anaerobic digestion using a substrate mixture of the organic fraction of MSW and primary sewage sludge (PSS). It also presents the development and application of a mathematical model of anaerobic digestion. The laboratory experiment shows that the mesophilic anaerobic co-digestion of a mixture of MSW and PSS is technically viable. The reduction in COD was 94%. The influent solids were 6.7% and the effluent solids were 3%. The maximum loading rate of 3.8 g VS/kg of reactor compares favorably with recent work by Norgaard (1995),

Table 2. Model parameter values

Y_a	Y_{va}	Y_m	K_{sm}	K_{am}	Y_{CO_2M}	Y_{CH_4}	K_{dm}	$\mu_{a,max}$	$\mu_{m,max}$
0.188	2.65	0.08	0.015	0.12	12.31	6.31	0.016	0.3	0.6

whose maximum loading rate varied from 3 to 5 g VS/kg of reactor per day. The production of biogas had a peak rate of 2.38 m³/m³ of reactor per day, which was more favorable than Norgaard (1995), whose biogas production was 1.3 m³/m³ of reactor per day. The experiment was predominantly stable during the run of 115 days, with output still improving at day 115.

The mathematical model was coded to simulate pH, NH₃ and CH₄ production from the experimental work. The model considers the inhibition caused by ammonia in the growth kinetics of methanogenic bacteria. Also, the model conserves the equilibrium between CO₂ and HCO₃⁻ and mass for CO₂, cations and NH₄⁺. The results of the model were shown to be satisfactory for simulating pH, NH₃ and CH₄.

As such, this experiment and modeling lends further credibility to the process of anaerobic digestion of the organic fraction of MSW co-digested with other waste streams. The significant loading rates, excellent COD reduction, sizeable gas production and stability of operation all lead to the desirability of exploiting anaerobic digestion for energy and environmental reasons.

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