Shortcut-Enhanced Quantum Thermodynamics

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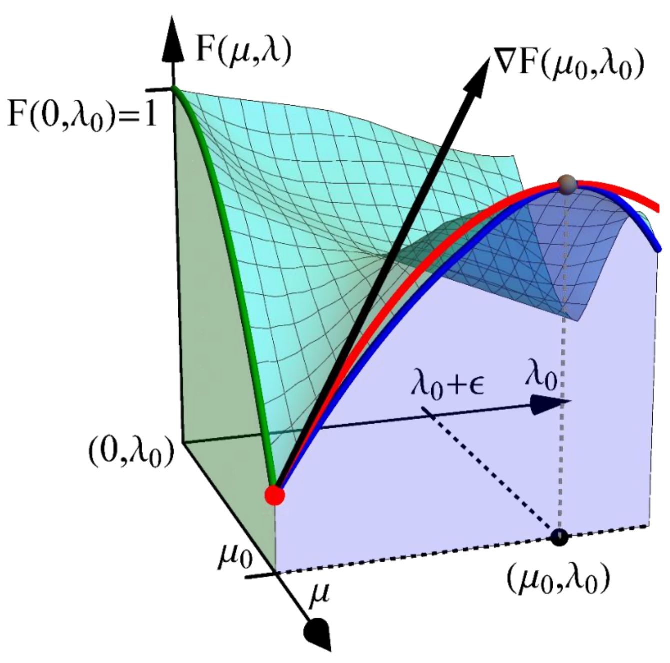
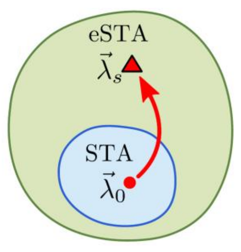
Abstract

In this project, we develop a novel route to overcome current problems with designing quantum technologies. The key idea is to combine shortcuts-to-adiabaticity and quantum thermodynamics. The potential for this merging is nearly self-explanatory: ideal thermodynamic processes are based on adiabatic processes which give maximum efficiency but unfortunately require infinite operation time; on the other hand, shortcuts-to-adiabaticity are techniques to speed-up adiabatic processes, e.g. the transport of Bose-Einstein condensate in anharmonic traps [1]. In line with this key idea, the technique of enhanced shortcuts to adiabaticity has been developed [2-3] and applied, for example, to spin squeezing in internal bosonic Josephson junctions [4]. In addition, quantum heat engines based on Bose-Einstein condensates have been studied. In detail, a novel quantum heat engine based on a spin-orbit and Zeeman-coupled Bose-Einstein condensate has been proposed. The work and heat involved as well as the associated efficiency have been discussed [5]. Moreover, quantum control of the dynamics of a classical piston coupled to a Rabi-coupled Bose-Einstein condensate has been developed and optimised; the work involved in this has also been examined [6].

Quantum Control via Enhanced Shortcuts to Adiabaticity[2-4]

Enhanced Shortcuts to Adiabaticity (eSTA): **Goal:** Design control scheme for quantum systems where STA cannot be applied directly. **Idea:** Merge Shortcuts to Adiabaticity (STA) with ideas from numerical optimal control (GRAPE algorithm) to design new analytical control schemes





Modification of control function:

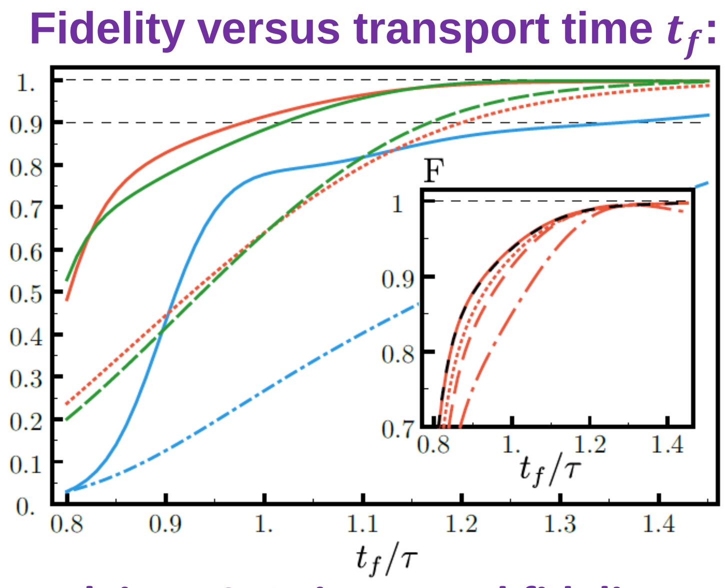
$$\vec{\varepsilon} = -\frac{\nabla F}{\hat{v}^T \mathrm{Hess}(F)\hat{v}} \approx -\frac{\vec{v}\|v\|^2}{\vec{v}^T \mathbb{H}\, \vec{v}}$$

Modification derived based on time-dependent perturbation theory while taking into account the properties of the STA solution for the ideal system

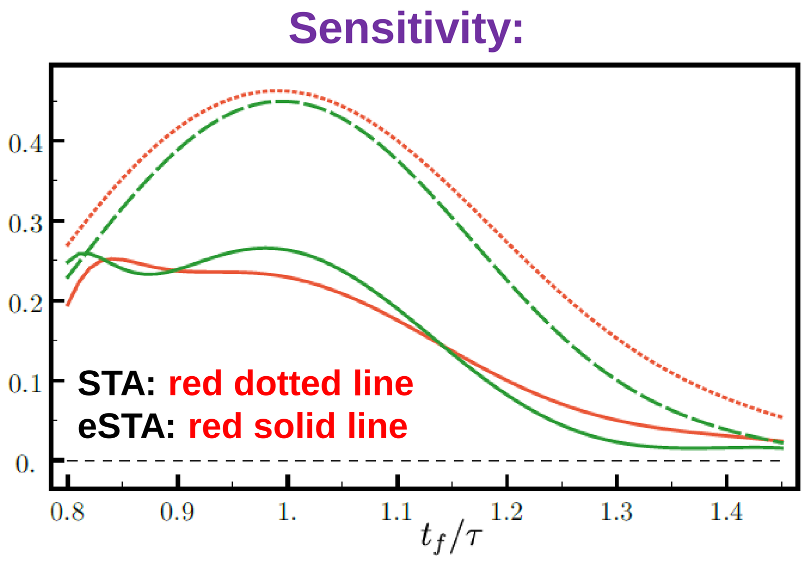
Important: the modification of the control function can be calculated only by using the STA scheme of the unperturbed system H_0!

Particle transport in traps and lattice potentials via eSTA[2-3]

Fidelity versus transport time t_f:



Sensitivity:

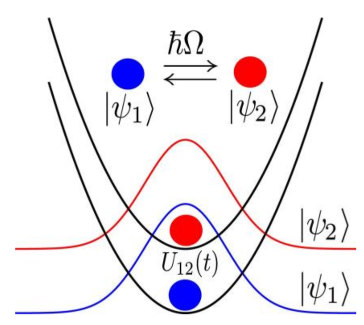


By applying eSTA: improved fidelity and improved robustness!

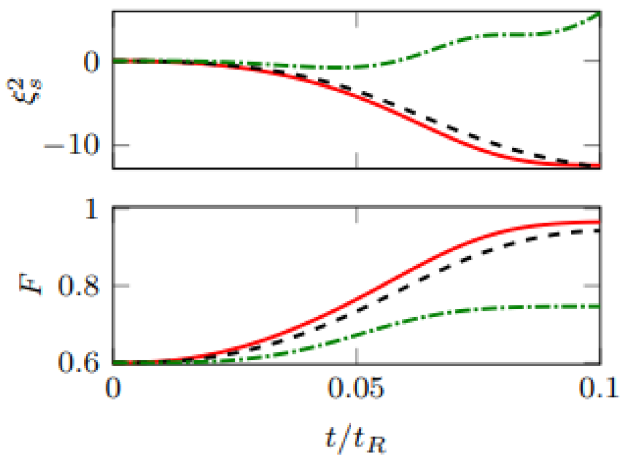
Spin squeezing in bosonic Josephson Junctions via eSTA[4]

Hamiltonian of an internal bosonic Josephson Junction:

$$H_{\mathrm{IBJJ}}(t) = U(t)\hat{J}_z^2 - \hbar\Omega\hat{J}_x$$



Coherent squeezing and Fidelity:



$$\xi_S^2(t) = \frac{N\Delta\hat{J}_z^2}{\langle\hat{J}_x\rangle^2} = \frac{\xi_N^2(t)}{\alpha^2(t)}$$

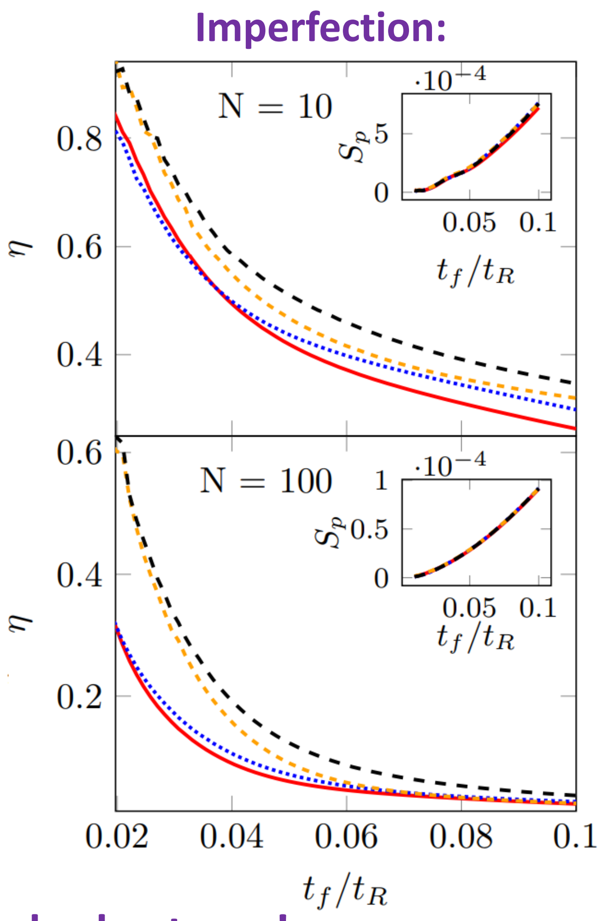
Imperfection:

$$\eta = \sqrt{(1-F)^2 + S_m^2 + S_t^2 + S_p^2}$$

STA: black, dashed line

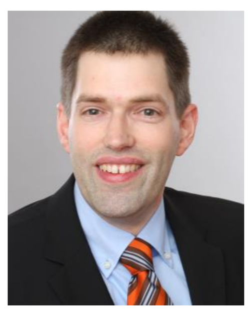
eSTA: red, solid line

Imperfection:



By applying eSTA: again, improved fidelity and improved robustness!

Group members



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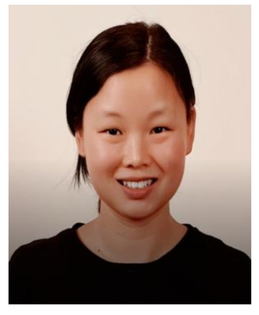

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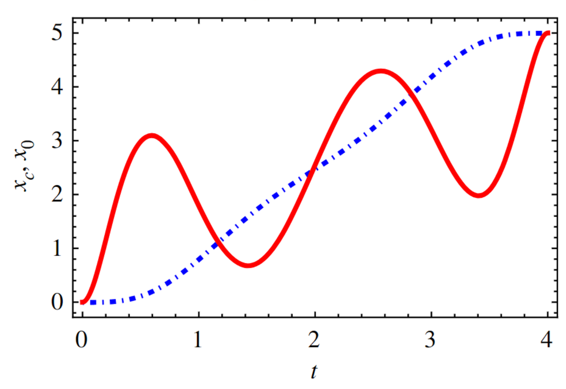
Transport of Bose-Einstein condensate (BEC) in anharmonic traps[1]

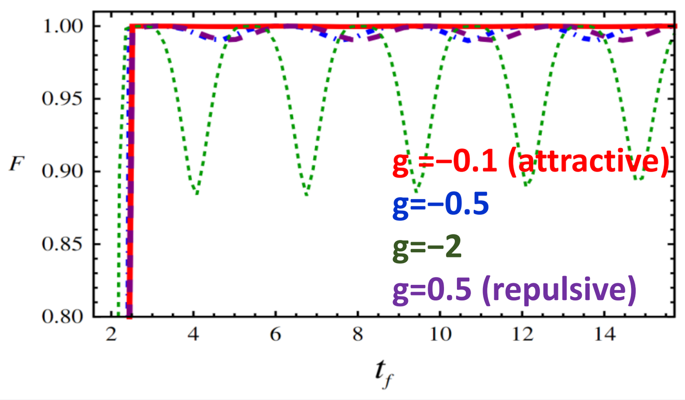
$$i\frac{\partial\psi(x,t)}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x,t) + g|\psi(x,t)|^2\right]\psi(x,t),$$

$$V(x,t) = \frac{1}{2}[x - x_0(t)]^2 + \frac{\kappa}{3!}[x - x_0(t)]^3 + \frac{\lambda}{4!}[x - x_0(t)]^4,$$



STA designed trap trajectory using variational method

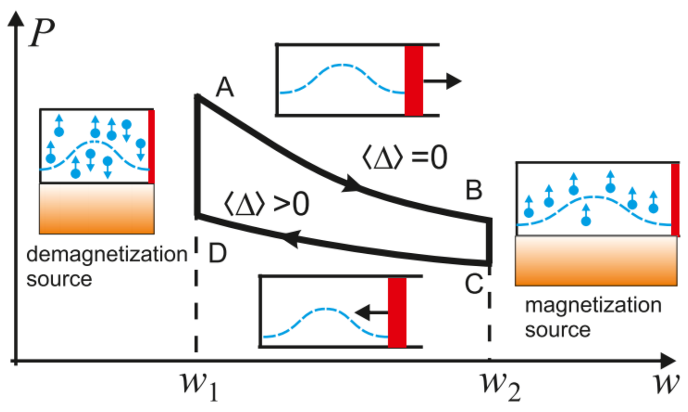
High Fidelity Transport of BEC



Quantum Thermodynamic Stirling cycle[5]

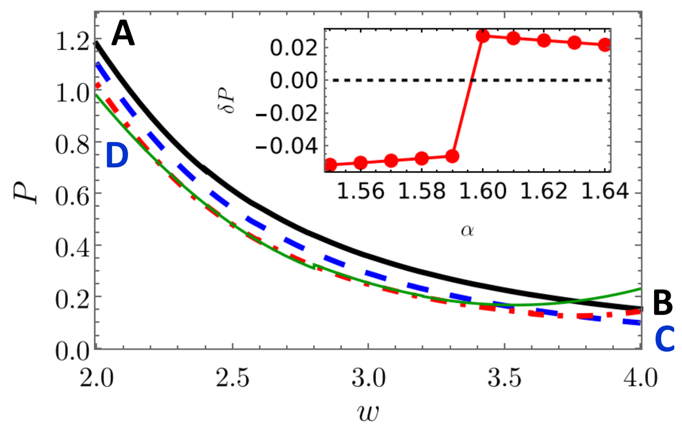
Working medium: **Bose-Einstein condensate (BEC) with spin-orbit and Zeeman coupling:**

$$\mathcal{H}_0 = \frac{\hat{p}^2}{2m}\hat{\sigma}_0 + \frac{\alpha}{\hbar}\hat{p}\hat{\sigma}_x + \frac{\hbar}{2}\Delta\hat{\sigma}_z + V(x)\hat{\sigma}_0$$



Key idea: Magnetization source: "cold thermal bath"
Demagnetization source: "hot thermal bath"

Stirling cycle:
A -> B: "Isothermal" expansion
B -> C: Isochore "cooling"
C -> D: "Isothermal compression
D -> A: Isochore "heating"



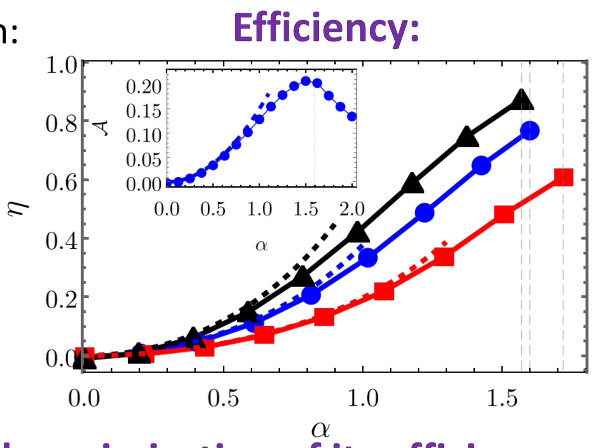
Expansion/Compression:
Work: $\langle W_e\rangle$, $\langle W_c\rangle$

"Heat": $\langle Q_h\rangle$

Efficiency:

$$\eta = \frac{\langle W_e\rangle + \langle W_c\rangle}{\langle Q_h\rangle}$$

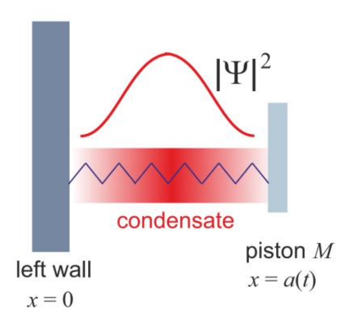
Efficiency:



New thermodynamic cycle with spin-orbit coupled BEC and maximisation of its efficiency

Quantum Control of classical piston motion[6]

Model: Rabi-coupled Bose-Einstein condensate coupled to a "classical" piston:



$$M\frac{d^2}{dt^2}a(t) = -M\Omega^2 a(t) + P(t)$$

$$P(t) = \left\langle \Psi(t)\middle|\frac{dV}{dz}(\hat{x} - a(t))\,\Psi(t)\right\rangle$$

$$\hat{H} = \hat{H}_B + \hat{H}_p + \hat{H}_{Bp}$$

Derivation of effective equations for BEC and piston:

$$i\hbar\frac{\partial}{\partial t}\psi_\uparrow = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{\hbar}{2}\Delta\cos\phi(t) + g_s|\psi_\uparrow|^2 + g_c|\psi_\downarrow|^2\right)\psi_\uparrow + [V_L(x) + V(x - a(t))]\,\psi_\uparrow - i\frac{\hbar}{2}\Delta\sin\phi(t)\,\psi_\downarrow$$

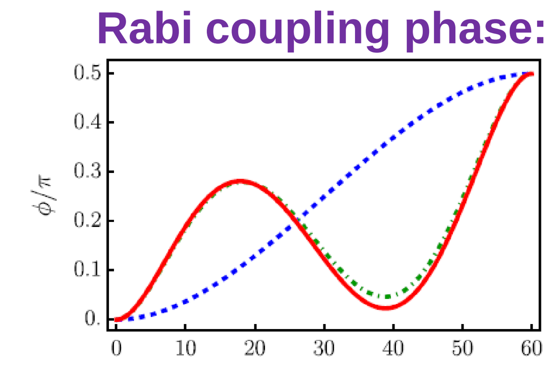
$$i\hbar\frac{\partial}{\partial t}\psi_\downarrow = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\hbar}{2}\Delta\cos\phi(t) + g_s|\psi_\downarrow|^2 + g_c|\psi_\uparrow|^2\right)\psi_\downarrow + [V_L(x) + V(x - a)]\,\psi_\downarrow + i\frac{\hbar}{2}\Delta\sin\phi(t)\,\psi_\uparrow$$

Mechanical work done by piston and its contributions:

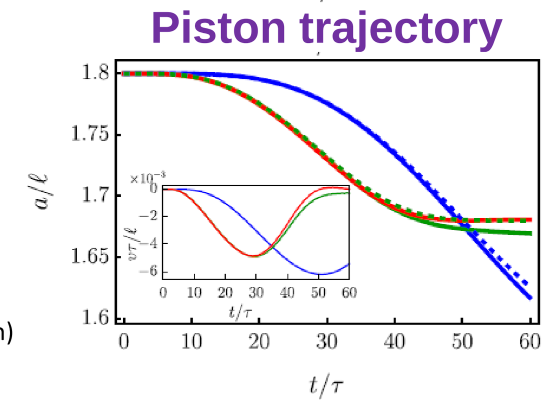
$$W_p = \int_0^{t_f} dt\,\frac{da}{dt}P(t) \qquad W_p = -\mathcal{E}_{\mathrm{st}} + W_{\phi,\mathrm{st}} + \delta W_p$$

(stationary energy change, stationary work related to Rabi coupling phase change, non-stationary contribution)

Rabi coupling phase:



Piston trajectory



Optimised: red, solid lines

Optimal design of the time-dependent direction of Rabi field to control position and velocity of the piston

References

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[4] M. Odelli, V. M. Stojanović and A. Ruschhaupt, Phys. Rev. Applied (2023), in print (see also arxiv.org/abs/2305.20032)

[5] J. Li, E. Ya Sherman and A. Ruschhaupt, Phys. Rev. A 106, L030201 (2022)

[6] J. Li, E. Ya Sherman and A. Ruschhaupt, submitted (see also arxiv.org/abs/2310.08675)

SFI Frontiers for the Future 19/FFP/6951
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