

Workshop on *Nonlinear Dispersive Waves*

University College Cork, April 24-25 2023

Aims:

This two-day international workshop will present recent research developments in the broad field of *Nonlinear Dispersive Waves*, with a particular emphasis on nonlinear waves arising in the ocean and atmosphere.

The workshop will operate on a hybrid basis: participants may attend either in person, or online. Registration is free, and interested participants should register directly with the organiser, Dr. David Henry, by emailing: d.henry@ucc.ie

Speakers:

Didier Clamond	<i>Université Côte d'Azur</i>
Adrian Constantin	<i>University of Vienna</i>
Olivia Constantin	<i>University of Vienna</i>
Joachim Escher	<i>Leibniz University Hannover</i>
Delia Ionescu-Kruse	<i>IMAR</i>
Rossen Ivanov	<i>TU Dublin</i>
David Lannes	<i>University of Bordeaux</i>
Bogdan Matioc	<i>University of Regensburg</i>
Emilian Parau	<i>University of East Anglia</i>
Jens Rademacher	<i>University of Hamburg</i>
Raphael Stuhlmeier	<i>University of Plymouth</i>
Samuel Walsh	<i>University of Missouri</i>

Sponsors:

This workshop is kindly supported by funding from the *School of Mathematical Sciences* (UCC), the *Irish Mathematical Society*, and *Science Foundation Ireland*.



School of Mathematical Sciences
Eolaíochtaí Matamaiticiúla



Workshop Programme

Nonlinear Dispersive Waves

University College Cork

April 24-25.

Schedule

The main venue for this workshop is room **G.01** in the **Western Gateway Building**, which is located just off UCC Main Campus (see the map on page 6). The talks on Monday afternoon, however, will take place in the **Dora Allman** room (with panoramic views of the campus and Cork city!) in the **Hub** building, which is located on the Main Campus.

	Monday	Tuesday
Venue:	WGB G.01	WGB G.01
9.00-9.15	<i>Registration/Coffee</i>	<i>Coffee</i>
9.15-9.30	<i>Opening remarks</i>	<i>Coffee</i>
9.30-10.10	Didier Clamond	David Lannes
10.10-10.50	Bogdan Matioc	Jens Rademacher
<i>10.50-11.20</i>	<i>Coffee</i>	<i>Coffee</i>
11.20-12.00	Rossen Ivanov	Delia Ionescu-Kruse
<i>12.00-14.00</i>	<i>Lunch</i>	<i>Lunch</i>
Venue:	Dora Allman room	WGB G.01
14.00-14.40	Joachim Escher	Adrian Constantin
14.40-15.20	Emilian Parau	Samuel Walsh
<i>15.20-15.50</i>	<i>Coffee</i>	<i>Coffee</i>
15.50-16.30	Olivia Constantin	Raphael Stuhlmeier

Titles and Abstracts

Didier Clamond (Université Côte d’Azur)

***Title:* On the recovery of rotational gravity waves from the seabed pressure**

Abstract: Considering a rotational steady bidimensional water wave with constant vorticity, we show how the free surface and the vorticity can be determined solely from the pressure at the seabed.

This is joint work with Joris Labarbe (U. Côte d’Azur) and David Henry (UCC).

Adrian Constantin (University of Vienna)

***Title:* Frictional effects in wind-driven ocean currents**

Abstract: Surface ocean currents have a significant influence on the climate and their dynamics depend to a large extent on the behaviour of the vertical eddy viscosity. We discuss an analytic approach to the study of wind-driven surface currents for general depth-dependent vertical eddy viscosities.

Olivia Constantin (University of Vienna)

***Title:* A complex analytic approach to some problems in fluid flows**

Abstract: In this talk we discuss several instances where methods from complex and harmonic analysis prove useful to specific problems arising in fluid mechanics.

We first consider irrotational periodic travelling waves and show the logarithmic convexity of certain flow quantities. As a by-product, we deduce, for instance, that the kinetic energy, the time-period of the particle paths, and the length of a streamline are larger near the surface and reduce with increasing depth. We then proceed in a different direction of investigation and obtain a complete solution to the problem of classifying all two-dimensional ideal fluid flows with harmonic Lagrangian labelling maps.

Some of these results are joint work with Maria Jose Martin.

Joachim Escher (Leibniz University Hannover)

***Title:* The Rayleigh–Taylor Condition for the Muskat Problem**

Abstract: Of concern is the moving boundary problem of a two-phase potential flow of two fluids with possible different densities and viscosities. Such problems are known as Muskat problems or two-phase Hele-Shaw flows. Due to the moving interfaces these problems are intrinsically nonlocal and highly nonlinear. A criterion is presented, known as the generalised Rayleigh–Taylor condition, which guarantees that for large classes of initial data these problems are classically well-posed, possibly on a finite time interval only. Away from the Rayleigh–Taylor regime the system becomes unstable and finger-shaped unstable

steady states can occur. A thin film approximation is also discussed. Here the dynamical behaviour is different: global weak solutions exist for any square integrable non-negative initial configuration. In addition, the flat steady state is globally stable in the class of weak solutions.

Delia Ionescu-Kruse (Institute of Mathematics of the Romanian Academy)

***Title:* On the short-wavelength stabilities of some geophysical flows**

Abstract: This talk is a survey of the short-wavelength stability method for rotating flows. This method is applied to the specific study of some exact geophysical flows. For Gerstner-like geophysical flows one can identify perturbations in certain directions as a source of instabilities with an exponentially growing amplitude, the growth rate of the instabilities depending on the steepness of the travelling wave profile. On the other hand, for certain physically realistic velocity profiles, steady flows moving only in the azimuthal direction, with no variation in this direction, are locally stable to the short-wavelength perturbations.

Rossen Ivanov (TU Dublin)

***Title:* Modelling internal waves over variable bottoms**

Abstract: We employ the Hamiltonian approach in analysing the effects of an uneven bottom on the internal wave propagation in the presence of stratification and underlying non-uniform currents. The presented models incorporate wave–current interactions and a variable bathymetry. A physical example is given by the equatorial internal waves in the presence of the Equatorial Undercurrent (EUC). The interface (physically coinciding with the thermocline and the pycnocline) satisfies in the long wave approximation a KdV–mKdV type equation with depth-dependent variable coefficients.

David Lannes (University of Bordeaux)

***Title:* Wave structure interaction in the Boussinesq regime**

Abstract: In this joint work with G. Beck and L. Weynans, we study the motion of a floating object in a fluid governed by the nonlinear dispersive Boussinesq equations. We show that the fluid+structure interaction problem can be reduced to an initial boundary value problem for the Boussinesq equations with nonstandard boundary conditions. We will comment on some mathematical aspects of this system and in particular insist on an hidden trace regularity phenomenon which plays a crucial role on the well-posedness theory; we will also show how this mathematical phenomenon can be exploited numerically, more or less in a similar way as the Riemann invariants for (nondispersive) hyperbolic systems such as the nonlinear shallow water equations. We will finally present some simulations.

Bogdan Matioc (University of Regensburg)

Title: Stratified Periodic Water Waves with Singular Density Gradients

Abstract: We discuss the existence of periodic free surface water waves that travel over a fluid whose density is stratified in a regime where both gravity and surface tension effects are included. The density of the fluid is assumed to be continuous, but its gradient maybe be unbounded. This setting is relevant when modeling ocean flows since the water density varies strongly only in thin layers called pycnoclines while it is almost constant in the layers between the pycnoclines.

This is joint work with Joachim Escher, Patrik Knopf, and Christina Lienstromberg.

Emilian Parau (University of East Anglia)

Title: A dissipative nonlinear Schrodinger model for wave propagation in the marginal ice zone

Abstract: Sea ice attenuates waves propagating from the open ocean. We propose to model the evolution of energetic unidirectional random waves in the marginal ice zone with a nonlinear Schrodinger equation, with a frequency dependent dissipative term consistent with current model paradigms and recent field observations. We also discuss shortly the scattering of waves by ice floes.

This is joint work with Alberto Alberello (UEA).

Jens Rademacher (University of Hamburg)

Title: Rotating convection with kinetic energy backscatter

Abstract: The so-called subgrid parameterisations by kinetic energy backscatter is widely used for large-scale geophysical flow simulations. It consists of damping by hyperviscosity and driving by negative viscosity to compensate lack of resolution and to stabilize the numerical scheme. Viewed as a PDE, we study the impact of backscatter via specific flows and waves for rotating Boussinesq and shallow water equations. On idealised domains we find that backscatter can generate equilibrium states as well as unboundedly growing flows and waves that take the form of explicit eigenmodes of the linear part in the nonlinear equations. Combined with bottom drag, in the shallow water case, we additionally find unusual spectra and bifurcations of geostrophic equilibria and inertia-gravity waves.

This is joint work with Artur Prugger (Bremen) and Jichen Yang (Zhuhai).

Raphael Stuhlmeier (University of Plymouth)

Title: A discrete Hamiltonian perspective on the classical instabilities of deep-water waves

Abstract: The stability of surface waves in deep water has traditionally been approached via linearisation, starting with various model equations, such as the NLS equation or (without restriction to narrow bandwidth) the Zakharov equation. Owing to the form of the deep-water dispersion relation, energy exchange among surface water waves - and thus the possibility of instability - occurs when four or more wavenumbers are involved. We shall employ the compact, Hamiltonian description of four-wave interaction due to Krasitskii.

In the usual mathematical sense, instability requires starting from a solution to a set of equations, and describes the evolution of perturbations to that solution. A handful of such explicit solutions - the monochromatic (Stokes') wave and bichromatic wave train - therefore form the backbone of classical instability results. The simplest, and best known is the Benjamin-Feir (BFI) or modulational instability, which will be presented in some detail. This instability involves three waves (one of which is counted twice) in near resonance, a carrier wave and two side bands. Two other instabilities have been studied extensively, and involve four distinct waves. Type Ia which begins with a bichromatic basic state and type Ib adds to the Benjamin-Feir instability an additional non-resonant satellite.

In all cases the entire non linear dynamics can be described by the level lines of a certain Hamiltonian function. This enables the explicit computation of steady state solutions, which have been the object of much recent interest. These steady state solutions characterise the linear instability of the underlying Stokes wave (for the BFI and type Ib) or the bichromatic sea state (Type Ia). The phenomenon of phase-locking (or phase coherence) is seen to be coincident with instability. In such cases the dynamic phase - a specific combination of the interacting phases - tends initially to a fixed value regardless of the initial configuration of the system. Moreover, certain heteroclinic orbits are identified as new discrete breather solutions, analogous to the famed Akhmediev breather.

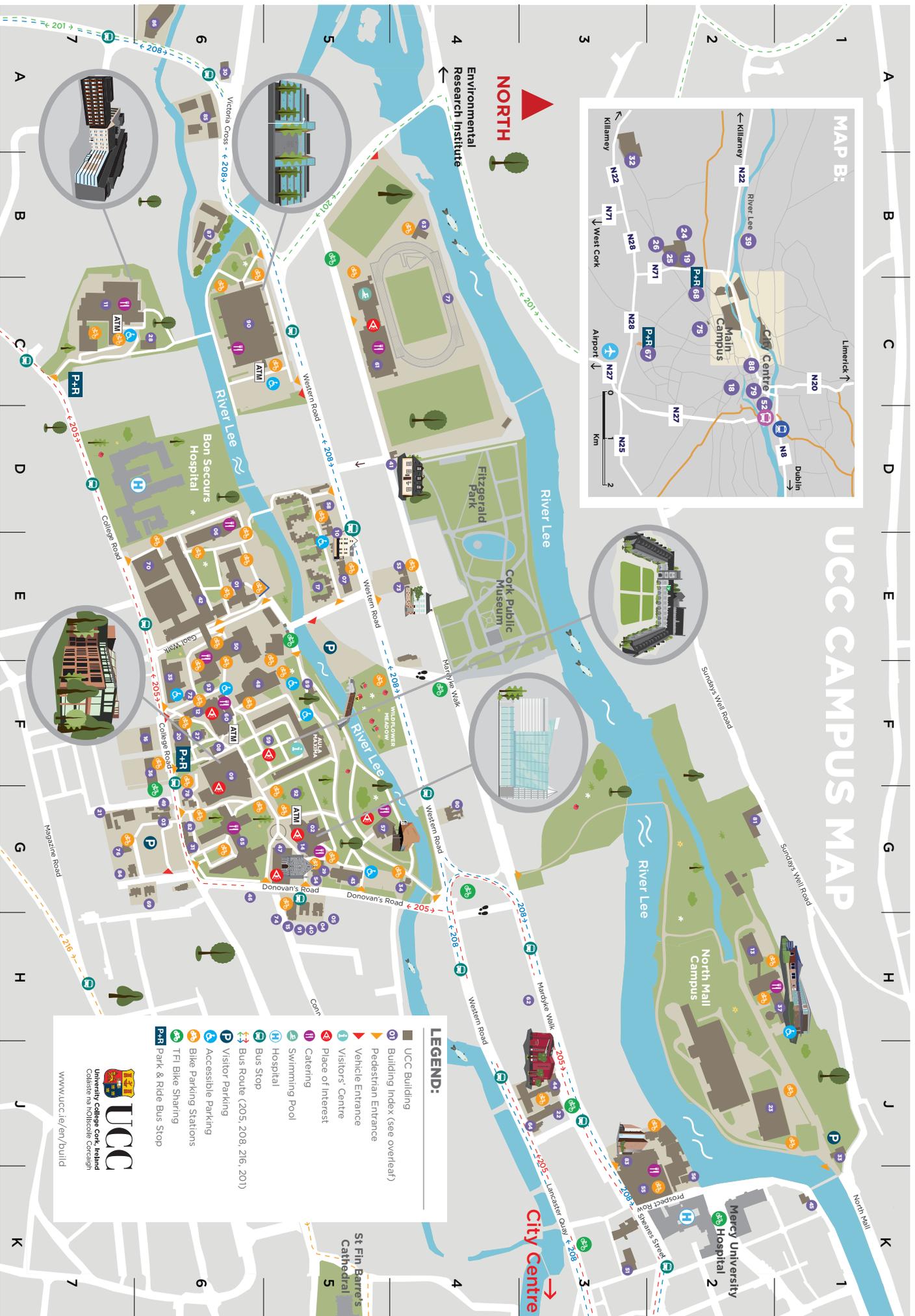
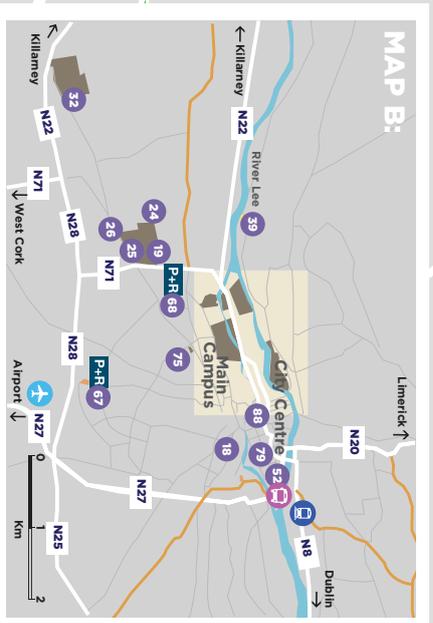
Samuel Walsh (University of Missouri)

***Title:* Desingularization and global continuation for hollow vortices**

Abstract: A hollow vortex is a region of constant pressure bounded by a vortex sheet and suspended inside a perfect fluid — think of it as a spinning bubble of air in water. In this talk, I will describe a general method for desingularizing non-degenerate translating, rotating, or stationary point vortex configurations into collections of steady hollow vortices. Through global bifurcation theory, moreover, these families can be extended to maximal curves of solutions that continue until the onset of a singularity. As specific applications, this machinery gives the first existence theory for co-rotating hollow vortex pairs and stationary hollow vortex tripoles, as well as a new construction of Pocklington's classical co-translating hollow vortex pairs. All of these families extend into the non-perturbative regime, and we obtain a rather complete characterization of the limiting behavior along the global bifurcation curve.

This is joint work with Ming Chen (University of Pittsburgh) and Miles H. Wheeler (University of Bath).

UCC CAMPUS MAP



Environmental Research Institute

LEGEND:

- UCC Building
- Building Index (see overleaf)
- ▶ Pedestrian Entrance
- ▶ Vehicle Entrance
- ▶ Visitors' Centre
- ▶ Place of Interest
- ▶ Catering
- ▶ Swimming Pool
- ▶ Hospital
- ▶ Bus Stop
- ▶ Bus Route (205, 208, 216, 201)
- ▶ Visitor Parking
- ▶ Accessible Parking
- ▶ Bike Parking Stations
- ▶ TFI Bike Sharing
- ▶ Park & Ride Bus Stop

University College Cork, Ireland

 Coláiste na hOllscoile Corcaigh

www.ucc.ie/en/build

