Applied Mathematics Seminar

UCC
University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

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On the Interaction of Rate-induced and Bifurcation-induced Tipping in Ecosystems

We discuss tipping phenomena in nonautonomous systems using an example of a bi-stable ecosystem model with environmental changes represented by time-varying parameters [Scheffer et al. *Ecosystems* 11 2008]. We give simple testable criteria for the occurrence of nonautonomous tipping from the herbivore-dominating equilibrium to the plant-only equilibrium using global properties of the autonomous frozen system with fixed-in-time parameters. To begin with, we use classical autonomous bifurcation analysis to identify a codimension-three degenerate Bogdanov-Takens bifurcation: the source of a dangerous subcritical Hopf bifurcation and the organising centre for bifurcation-induced tipping (B-tipping). Then, we introduce basin instability analysis to identify parameter paths along which genuine nonautonomous rate-induced tipping (R-tipping) occurs without crossing any classical autonomous bifurcations.

We explain nonautonomous R-tipping in terms of maximal canard trajectories, and produce nonautonomous tipping diagrams in the plane of the magnitude and rate of a parameter shift to uncover intriguing R-tipping tongues and wiggling tipping-tracking bifurcation curves. Discussion of non-trivial dynamics arising from the interaction between B-tipping and R-tipping identifies "points of no return" where tipping cannot be prevented by the parameter trend reversal and "points of return tipping" where tipping is inadvertently induced by the parameter trend reversal. Our results give new insight into the sensitivity of ecosystems to the magnitudes and rates of environmental change. Finally, a comparison between 'tilted' saddle-node and subcritical Hopf normal forms reveals some universal tipping properties due to basin instability, a generic dangerous bifurcation, or the combination of both.

Thursday, 12.03.2020 · 12:00 · WGB G02
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