UCC
University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

School of Mathematical Sciences Student Summer Internship Programme 2022

List of projects

(i) **Advances in Machine Learning Applicable to Production and Research & Development in the Pharmaceutical Industry**

(ii) **Mathematical aspects of atmospheric flows**

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Advances in Machine Learning Applicable to Production and Research & Development in the Pharmaceutical Industry

This project will develop an inventory of machine learning techniques, both established classical methods in the area of statistics and more recent advances at the interface of the areas of statistics and computer science. The inventory will include a high level description of the methods in non-technical language. The second stage of the project will demonstrate how these methods can be applied in the pharmaceutical industry. Examples drawn from statistical, machine learning and pharmaceutical academic literature as well as pharmaceutical regulatory agency guidelines will be used. Data collected for both monitoring/assessment of production of pharmaceuticals and research & development of investigational pharmaceuticals for regulatory submission will be considered.

Supervisor: Dr Michael Cronin

Mathematical aspects of atmospheric flows

The study of large-scale atmospheric flows is a fascinating subject which poses a wide range of challenges. Indeed, several mathematical advances and breakthroughs were directly inspired by the quest to develop an improved knowledge of meteorology, for instance. Aside from being theoretically challenging, it is a subject of massive importance, being the basis for understanding many weather- and climate-related phenomena.

This project will examine mathematical methods and approaches which provide a theoretical framework underpinning various meteorological phenomena. Please contact d.henry@ucc.ie for further information, or for informal discussions relating to the project.

Supervisor: Dr David Henry

The Fourier, Radon and X-ray transforms

The aim of this project is to study fundamental properties of the Fourier, Radon and X-ray transforms, their inversion formulas and a survey of applications of the latter two. The starting point will be an in-depth study of the Fourier transform using [1], especially Chapters 5 and 6. Following that, the Radon and X-ray transforms will be considered, their interplay with the Fourier transform, and inversion formulas will be obtained. Finally, the project will focus on the X-ray and restricted X-ray transforms and will look at different line complexes and the question of whether they are admissible or not, i.e. whether the associated restricted X-ray transform can be inverted for the line complex in question.

References

[1] E. M. Stein, R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, 2003.

Supervisor: Dr Spyridon Dendrinos

Bump-and-revalue: estimating the Greeks

The seller ("writer") of an option is exposed to market risk for the lifetime of the option. For most options the position must be actively managed by a strategy of dynamic hedging in order to mitigate against that risk. Traders, to whom the task falls, need knowledge of certain hedge parameters, option price sensitivities known as the Greeks. So it is not sufficient to be able to value an option, we must also be able to calculate how its value will change with changes in the price of the underlying asset, volatility, interest rates etc. For most options, these quantities have no closed form and must be estimated.

Since the Greeks are characterised by first- and second-order partial derivatives of the option value, we can use finite difference approximation in combination with Monte Carlo estimation to approximate their values numerically. This is the so-called "bump-and-revalue" method, and the project will look at the theory, practice, and limitations associated it for a variety of derivative securities and asset price models.

Those interested in applying should have successfully completed the module MF3052 Derivatives, Securities, and Option Pricing. Python will be used for computational work.

Supervisor: Dr Conall Kellly

Minimal Surfaces

The study of minimal surfaces started in the 18th century and has grown into a vast field that intersects many branches of mathematics; it has found important applications in geometry, topology and theoretical physics. Indeed, in one of the theory's great triumphs, minimal submanifolds and minimal submanifold techniques were used to give the first rigorous proof of the positive mass theorem from general relativity. Minimal surfaces also played a significant role in the development of the calculus of variations, non-linear elliptic partial differential equations and geometric measure theory, by providing motivating examples and important models. The theory of minimal submanifolds is both beautiful and rich, with many open questions remaining even in the classical setting of minimal surfaces in Euclidean 3-space.

Two of many books on the subject:

(i) Colding and Minicozzi: 'A course in minimal surfaces'
(ii) Osserman: 'A survey of minimal surfaces'

Some prior knowledge of complex analysis, and differential geometry are necessary for working on this project. Please get in touch with Martin Kilian at m.kilian@ucc.ie for further details.

Supervisor: Dr Martin Kilian

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Investigating the effects of delayed feedbacks on the shutdown of the Atlantic Meridional Overturning Circulation

There is increasing evidence that the ocean current that drives the Gulf Stream, the Atlantic Meridional Overturning Circulation (AMOC), is approaching a so-called tipping point, at which it will shut down and no longer provide Western and Northern Europe with its relatively warm climate [1, 2]. A detailed understanding of the changes that would take place in the ocean as AMOC approaches this tipping point requires the study of both sophisticated forecasting models and conceptual models that are amenable to rigorous mathematical analysis.

In this project the student will study the dynamics of a conceptual ocean model that is designed to capture important dynamical features of AMOC [3]. The focus will be on understanding how physical features of the ocean, modelled as delayed feedbacks, affect the behaviour of the system near its tipping point. This project will suit a student with strengths in dynamical systems theory and numerical methods, as well as an interest in mathematical modelling and delay differential equations.

[1] Caesar, L., Rahmstorf, S., Robinson, A., Feulner, G., & Saba, V. (2018). Observed fingerprint of a weakening Atlantic Ocean overturning circulation. Nature, 556(7700), 191-196.
[2] Boers, N. (2021). Observation-based early-warning signals for a collapse of the Atlantic Meridional Overturning Circulation. Nature Climate Change, 11(8), 680-688.
[3] Keane, A., Pohl, A., Dijkstra, H. A., & Ridgwell, A. (2022). A simple mechanism for stable oscillations in an intermediate complexity Earth System Model. arXiv preprint arXiv:2201.07883.

Supervisor: Dr Andrew Keane

Multiverse Analysis in Agricultural Science

A problem that is overlooked in the analysis of data is the different decisions made in data filtration, variable selection and model building, and the impact these decisions have on results and conclusions. Most of research today involves modelling data with large numbers of candidate variables and/or incomplete information. Consequently, researchers make many decisions during analysis that influence their results and conclusions. Often these decisions are made with no proper assessment of their impact. There are many plausible decisions that could be made during data analysis and ignoring the impact of these decisions can lead to work that is not reproducible. One approach for taking this aspect of data analysis into account is multiverse analysis. This analytical technique offers a transparent and systematic reporting of alternative results that could be obtained from plausible decisions made during data analysis. The assumption is that for a given data set, there is a multiverse, multiple universes of analysis. In multiverse analysis all "reasonable" analytic decisions are evaluated in parallel and interpreted collectively. Multiverse analysis gives an assessment of how much conclusions change because of decisions made in data analysis. It also provides indications about which choices have most impact on the results. So it provides transparency about the results presented and an assessment of robustness. Unsurprisingly, a major issue is how to present the findings from a multiverse analysis, that is, how to visualise multiverse analysis. Researchers need tools to support the exploration of the multiverse as they are no longer dealing with a single analysis, but potentially hundreds of alternatives arising from possible combinations of analytic decisions. Although multiverse analysis has most commonly been used in psycho-social research, there is little evidence of its application in other context. This project will explore the application of multiverse analysis in an agricultural science context.

Supervisor: Kathleen O'Sullivan

Critical Transitions

Critical transitions or tipping points are strongly nonlinear phenomena which can be described in layman's terms as large, sudden, and often unexpected changes in the state of a nonlinear system, caused by small and slow changes in the external inputs. The main challenge is that critical transitions cannot, in general, be understood in terms of the classical autonomous bifurcation theory. Thus, they require an alternative approach.

This project will combine non-autonomous stability theory, geometric singular perturbation theory, compactification techniques, and numerical analysis to study critical transitions in one of the application areas:

(i) Climate dynamics.

(ii) Ecological dynamics.

(iii) Evolutionary games.

I invite interested students to discuss the project in more detail.

Supervisor: Prof. Sebastian Wieczorek