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Acquiring Local Preferences of Weighted Partial MaxSAT

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Abstract

Many real-life problems can be formulated as boolean satisfiability (SAT). In addition, in many of these problems, there are some hard clauses that must be satisfied but also some other soft clauses that can remain unsatisfied at some cost. These problems are referred to as Weighted Partial Maximum Satisfiability (WPMS). For solving them, the challenge is to find a solution that minimizes the total sum of costs of the unsatisfied clauses. Configuration problems are real-life examples of these, which consist on customizing products according to user's specific requirements. In the literature, there exist many efficient techniques for finding solutions having minimum total cost. However, less attention has been paid to the fact that in many real-life problems the associated weights for soft clauses can be unknown. An example of such situations is when the users cannot provide local preferences but instead express global preferences over complete assignments. In these cases, the acquisition of preferences can be the key to finding the best solution. In this paper, we propose a method to formalize the acquisition of local preferences. The process can be done by solving the associated system of linear equations of a set of complete assignments and their cost with Linear Algebra techniques. Furthermore, we formalize the characteristics and size of the complete assignments required to be able to acquire all the local weights. We have formalized such problem as well. We also present an heuristic algorithm that searches for such assignments, which performs promisingly on many benchmarks from the literature.

Background

The challenge of solving many real-life problems is beyond finding a satisfiable assignment due to the preferences specified within the problem. Generally, the users specify certain preferences for certain configurations and therefore the objective becomes finding the best solution, which is called the optimal solution. This solution maximizes the satisfiability of the specified preferences, i.e. it minimizes the cost of their violation.
Soft constraints allow users to express their local preferences over constraints, variables and/or tuples; therefore their use has been the traditional way to model such problems. However, in more recent works (e.g. [1]) a new way of expressing preferences is considered. In this paper the users express preferences in a general manner: over some complete assignments. Then, it is not clear how to model such general preferences locally.
We would like to recall that the objective of solving problems with preferences is to find the optimal solution. For this reason, it is very important to acquire local preferences with the motivation to be able of finding the optimal solution of the problem. In this context, this paper focuses in the local preferences acquisition task.
The Weighted Partial MaxSAT problem (WPMS) allows us to express local preferences since it incorporates weights associated with soft clauses. Then, by means of the weights associated to the soft constraints, the users can fix the preferences locally, that is to say, over each soft constraint. Each weight is the penalization associated with a solution that does not satisfy such clause. The higher is the weight of a clause, the higher is the preference of the satisfaction of such clause. The objective function of such problems is to minimize the sum of the weights of the unsatisfied soft clauses.
We have formalized the acquisition of local preferences for SAT problems in order to be able to model them as WPMS. We have approached this task by modeling the problem as a system of linear equations and subsequently using Linear Algebra techniques to solve the resultant system. Furthermore, we are interested on the characteristics and size of the complete assignments, and their total costs, required to acquire all local weights.

Preliminaries

We deal with the task of acquiring local preferences in SAT. These local preferences are subsequently modelled as Weighted Partial MaxSAT (WPMS). For this reason we provide a formal definition of such models.
The SAT and MaxSAT problems are well known to be NP-hard [2, 3]. In SAT for a given boolean formula the objective is to find an assignment that satisfies all the clauses. In MaxSAT the objective is to find an assignment that maximizes the number of satisfied clauses.
A CNF is a set of clauses $(C_1, C_2, ..., C_m)$ in conjunctive normal form where C_i is a disjunction of literals. A literal is a boolean variable x or its negation $\overline{x}$ [4]. The SAT problem is finding an assignment $var(f) \rightarrow \forall C_i \in f, C_i \rightarrow True$. Let ϕ be a function such that $\phi(C) \rightarrow 1$ iff $C \rightarrow True$, otherwise $\phi(C) \rightarrow 0$. The MaxSAT is finding a assignment to satisfy maximum number of the clauses: $var(f) \rightarrow max(\sum_1^m \phi(C_i))$.
The following notation was described in [5]. The weighted partial MaxSAT (WPMS) is composed of hard clauses and soft clauses with weights. Each weight associated to each soft clause C_i is denoted as w_i. Then, a weighted partial clause is a pair (C_i, w_i), where w_i is natural number. The formula of Weighted Partial MaxSat problem is

$$\varphi = \{(C_1, w_1), ..., (C_m, w_m), ..., (C_{m+1}, \infty), ..., (C_{m+m'}, \infty)\}$$

where the first m clauses are soft and the last m' are hard. The weight of hard clauses should be ∞ compared to soft ones. $var(\varphi), var(C)$ denotes the set of variables in φ and C respectively. I is a function that gives the truth assignment for literals, clauses, SAT formulates. Then, $I : var(\varphi) \rightarrow \{0, 1\}$ is noted as the assignment for φ. $cost(\varphi) = \sum_{i=1}^m w_i(1 - I(C_i))$. Then, w_i is summed to the total cost iff clause C_i is unsatisfied. The objective function of the WPMS is to minimize $cost(\varphi)$, that is to say, to minimize the total cost associated to the unsatisfied clauses.

Formalization

we intend to acquire the unknown weights of the soft clauses of a WPMS. Without loss of generality and with the purpose of simplifying notation, we consider that all the weights of the m soft clauses of the WPMS are unknown. We use the fact that, by definition, the relation between w and $cost(\varphi)$ is linear for composing a system of linear equations that allow us to acquire the unknown weights.
For solving this system of linear equations we use Gaussian elimination [6, 7], which performs elementary row reductions by performing operations over the coefficient matrix. With the first part of the process of Gaussian elimination the original matrix of coefficients is converted into its row echelon form. The second part of the algorithm continues the row reduction until its convergence to a row reduced echelon form, in order to find the values of the undetermined variables, i.e. to find the solution of the system of linear equations.

Experiments

The objective of our experiments was to evaluate the effectiveness of our algorithm in finding the full set of m LI solutions and subsequently using them for acquiring the m unknown weights. In our experiments, we used SCIP as SAT solver for finding the solutions that satisfy the hard clauses. As previously mentioned, after each iteration, we added or updated the clauses in the SAT instance in order to find a new LI solution. We set up an stop-condition to terminate the search: an iteration-limit (for these experiments: $\#NoNewSolIters > \#SoftClaus - \#LISols$).
We performed our experiments on a set of benchmarks taken from the MaxSAT Evaluations competitions. These datasets are either from `random`, `crafted` or `industrial` of the `Weighted Partial MaxSAT` catalog of MaxSAT Evaluations competitions 2006. Table 1 shows the benchmarks analyzed in ascending order by the number of total clauses in the dataset. Column `#Total Claus` represent the number of total clauses in the dataset. The hard clauses and soft clauses in the column `#Hard Claus` and `#Soft Claus` respectively. (Here, we assume that all the soft clauses have unknown weights). The number of variables is represented as `#Vars`. In the `Result` part of the Table 1, we listed the number of LI solutions found, the number of acquired unknown weights, runtime and the number of iterations in experiment.

Results

Table: Benchmarks and results obtained.

Benchmark Details					Result			
Benchmark	#Total Claus	#Hard Claus	#Vars	#Soft Claus	#LI Sols	#ACQ Claus	Time(s)	#Iters
8.wcsp.log	25	17	12	8	7	7	<1	15
8.wcsp.dir	29	21	20	8	**8**	**8**	<1	**8**
cat_paths_60_70_0001.txt	397	324	73	73	**73**	**73**	271	74
cat_paths_60_70_0004.txt	442	372	70	70	**70**	**70**	150	73
54.wcsp.log	479	412	96	67	**67**	**67**	171	**67**
54.wcsp.dir	508	441	154	67	41	13	193	109
1502.wcsp.log	534	325	311	209	205	201	6541	415
cat_paths_60_70_0003.txt	540	470	70	70	**70**	**70**	144	**70**
cat_paths_60_70_0007.txt	551	477	74	74	**74**	**74**	249	**74**
cat_paths_60_80_0002.txt	553	472	81	81	**81**	**81**	390	**81**
cat_paths_60_70_0006.txt	566	495	71	71	**71**	**71**	142	**71**
cat_paths_60_70_0000.txt	598	528	70	70	**70**	**70**	137	74
cat_paths_60_70_0005.txt	605	533	72	72	**72**	**72**	111	**72**
cat_paths_60_70_0002.txt	610	540	70	70	65	19	111	136
cat_paths_60_80_0001.txt	612	531	81	81	**81**	**81**	292	**81**
cat_paths_60_80_0003.txt	615	535	80	80	**80**	**80**	272	**80**
cat_paths_60_80_0006.txt	633	551	82	82	**82**	**82**	283	**82**
1502.wcsp.dir	636	427	515	209	81	15	1655	291
29.wcsp.log	692	610	101	82	**82**	**82**	300	**82**
29.wcsp.dir	711	629	139	82	60	14	476	143
503.wcsp.log	934	791	201	143	**143**	**143**	5496	171
404.wcsp.log	1037	937	129	100	**100**	**100**	619	**100**
404.wcsp.dir	1066	966	187	100	99	97	737	199
cat_paths_60_140_0002.txt	1833	1692	141	141	**141**	**141**	3436	143
cat_paths_60_130_0000.txt	1936	1806	130	130	129	66	2056	391
cat_paths_60_140_0003.txt	1954	1810	144	144	**144**	**144**	5395	179
42.wcsp.log	2016	1826	247	190	**190**	**190**	10132	328
cat_paths_60_140_0001.txt	2024	1881	143	143	**143**	**143**	4697	162
cat_paths_60_150_0003.txt	2029	1878	151	151	**151**	**151**	5254	**151**
42.wcsp.dir	2073	1883	361	190	177	152	7836	368
cat_paths_60_170_0005.txt	2089	1917	172	172	**172**	**172**	9807	286
cat_paths_60_150_0002.txt	2092	1942	150	150	**150**	**150**	6072	202
cat_paths_60_150_0000.txt	2097	1947	150	150	**150**	**150**	4618	193
cat_paths_60_170_0003.txt	3016	2845	171	171	**171**	**171**	7950	**171**
408.wcsp.log	3149	2949	264	200	**200**	**200**	18228	220
505.wcsp.dir	3536	3296	552	240	**240**	**240**	29540	341
cap61.wcsp	3978	3164	1664	814	47	10	17985	861
cap62.wcsp	3978	3164	1664	814	48	11	17883	862
cat_sched_60_160_0000.txt	5314	5154	160	160	143	52	1303	304
cap81.wcsp	6273	5000	2600	1273	31	11	26866	1304
cap82.wcsp	6273	5000	2600	1273	30	13	27113	1304
cap101.wcsp	6273	5000	2600	1273	37	17	27154	1310
cap102.wcsp	6273	5000	2600	1273	40	22	27277	1313
1504.wcsp.log	6593	5988	929	605	232	2	176013	488

Future Work

We will work on the case if the users can only give the rank of preference to the solutions. We will also work on a more restrictive variant of the problem described in this problem, in which we can only compute k solutions whose associated equations are linearly independent, where $k < m$. Therefore, we want to find the k solutions that maximize the acquisition of weights.

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