

MATHEMATICS DEPARTMENT UNIVERSITY COLLEGE CORK

IRISH MATHEMATICS INTERVARSITY 2005
UNIVERSITY COLLEGE CORK

Professor Des MacHale

Time allowed: 3 Hours

Calculators and tables may be used

Answer all ten questions

- If ABC is a triangle such that $\cos 3A + \cos 3B + \cos 3C = 1$, find, to the nearest degree, the size of the largest angle of the triangle.
- Show how to write down a hundred consecutive positive integers none of which is a prime or a power of prime.
- If $S_r = \sum_{i=1}^n i^r$, express $S_5 + S_7$ in terms of S_1 only.
- Evaluate $\lim_{\theta \rightarrow \frac{\pi}{2}} (\sin \theta)^{\tan \theta}$.
- Find any positive integers x, y and z such that $x^5 + y^6 = z^7$.
- You are given a finite number of points in the plane, no two of which are the same distance apart. Each point is joined by a straight line segment to the point nearest to it. Show that no point is joined to more than five other points.
- A plane quadrilateral has the properties that a circle passes through its four vertices and another circle touches its four sides internally at A, B, C and D . Prove that AC is perpendicular to BD .
- Evaluate the indefinite integral $\int \frac{dx}{\sqrt[4]{1+x^4}}$
- A piece of wire of length K is cut into two pieces. The first piece is bent to form a square of side s and the second piece is bent to form a circle of radius r . If the sum of the areas of the circle and the square is as small as possible, find the value of s/r .
- What is the minimum number of pieces into which this figure must be cut, so that these pieces can be reassembled to form a square?

