The Conditional Pricing of Systematic and Idiosyncratic Risk in the UK Equity Market

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Abstract

We test whether firm idiosyncratic risk is priced in a large cross-section of U.K. stocks. A distinguishing feature of our paper is that our tests allow for a conditional relationship between systematic risk (beta) and returns, i.e., conditional on whether the excess market return is positive or negative. We find strong evidence in support of a conditional beta/return relationship which in turn reveals conditionality in the pricing of idiosyncratic risk. We find that idiosyncratic volatility is significantly negatively priced in stock returns in down-markets. Although perhaps initially counter-intuitive, we describe the theoretical support for such a finding in the literature. Our results also reveal some role for liquidity, size and momentum risk but not value risk in explaining the cross-section of returns.

JEL Classification: G11; G12.

Key Words: asset pricing; idiosyncratic risk; turnover; conditional beta.

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1. Introduction

Idiosyncratic, or non-systematic, risk arises due to asset price variation that is specific to a security and is not driven by a systematic risk factor common across securities. It is typically estimated using a pricing model of returns with common risk factors and obtained as the residual unexplained variation. In this paper we revisit the question of whether idiosyncratic risk is priced in a large cross-section of U.K. stocks. A distinguishing feature of our paper is that we incorporate a conditional relationship between systematic risk (beta) and return in our tests for which we find strong evidence. This in turn reveals conditionality in the pricing of idiosyncratic risk. We control for other stock risk characteristics such as liquidity (which we decompose into systematic and idiosyncratic liquidity), size, value and momentum risks which may explain some idiosyncratic risk.

The role of idiosyncratic risk in asset pricing is important as investors are exposed to it for a number of reasons, either passively or actively. These include portfolio constraints, transaction costs that need to be considered against portfolio rebalancing needs or belief in possessing superior forecasting skills[1]. Assessing if and how idiosyncratic volatility is priced in the cross-section of stock-returns is relevant in order to determine if compensation is earned from exposure to it. Controlling for systematic risk factors and other stock characteristics, if the expected risk premium for bearing residual risk is positive, it may support holding idiosyncratic difficult-to-diversify stocks and other undiversified portfolio strategies. Conversely, negative pricing of idiosyncratic risk clearly points to increased transaction costs to achieve a more granular level of portfolio diversification to offset it. Idiosyncratic risk is important and large in magnitude, and accounts for a large proportion of total portfolio risk.[2] A better characterization of it will improve the assessment of portfolio risk exposures and the achievement of risk and return objectives.

[1] Portfolio constraints include the level of wealth, limits on the maximum number of stocks held or restrictions from holding specific stocks or sectors. Funds with a concentrated style willingly hold a limited number of stocks. Even large institutional portfolios benchmarked to a market index typically hold a subset of stocks and use techniques to minimize non-systematic exposures.

[2] Campbell et al. (2001) for a US sample find firm-level volatility to be on average the largest portion (over 70%) of total volatility, followed by market volatility (16%) and industry-level volatility (12%). Our results are broadly consistent, with the firm-level component accounting on average for over 50% of total variance, with the rest evenly split between the market and industry components.

Traditional pricing frameworks such as the CAPM imply that there should be no compensation for exposure to idiosyncratic risk as it can be diversified away in the market portfolio. However, this result has been challenged both theoretically and empirically. Alternative frameworks relax the assumption that investors are able or willing to hold fully diversified portfolios and posit a required compensation for idiosyncratic risk. Merton (1987) shows that allowing for incomplete information among agents, expected returns are higher for firms with larger firm-specific variance. Malkiel and Xu (2002) also theorise positive pricing of idiosyncratic risk using a version of the CAPM where investors are unable to fully diversify portfolios due to a variety of structural, informational or behavioural constraints and hence demand a premium for holding stocks with high idiosyncratic volatility. In empirical testing several studies find a significantly positive relation between idiosyncratic volatility and average returns; Lintner (1965) finds that idiosyncratic volatility has a positive coefficient in cross-sectional regressions as does Lehmann (1990) while Malkiel and Xu (2002) similarly find that portfolios with higher idiosyncratic volatility have higher average returns.

However, the direct opposite perspective on the pricing of idiosyncratic risk, that of a negative relation between idiosyncratic volatility and expected returns, has also been theorised and supported by empirical evidence. One theory links the pricing of firm idiosyncratic risk to the pricing of market volatility risk. Chen (2002) builds on Campbell (1993 and 1996) and Merton's (1973) ICAPM to show that the sources of assets' risk premia (risk factors) are the contemporaneous conditional covariances of its return with (i) the market, (ii) changes in the forecasts of future market returns and (iii) changes in the forecasts of future market volatilities. In particular, this third risk factor, which the model predicts has a negative loading, indicates that investors demand higher expected return for the risk that an asset will perform poorly when the future becomes less certain, i.e., higher (conditional) market volatility[3]. Ang et al (2006) argue that stocks with high idiosyncratic volatilities may have high exposure to market volatility risk, which lowers their average returns, indicating a negative pricing of idiosyncratic risk in the

[3] Conversely, assets with high sensitivities (covariance) to market volatility risk provide hedges against future market uncertainty and will be willingly held by investors, hence reducing the required return.

cross-section. If market volatility risk is a (orthogonal) risk factor, standard models of systematic risk will mis-price portfolios sorted by idiosyncratic volatility due to the absence of factor loadings measuring exposure to market volatility risk. In empirical testing on US data, Ang et al. (2006) find that exposure to aggregate volatility risk accounts for very little of the returns of stocks with high idiosyncratic volatility (controlling for other risk factors) which, they say, remains a puzzling anomaly[4]. We add to this literature by investigating the pricing of idiosyncratic volatility in a large sample of U.K. stocks in conditional market settings and controlling for other risk factors and stock characteristics in the cross-section.

Much like the mixed theoretical predictions concerning the pricing of idiosyncratic risk, empirical findings around the idiosyncratic volatility puzzle (negative relation between idiosyncratic risk and returns) are also quite mixed. For instance, Malkiel and Xu (2002), Chua et al.(2010), Bali an Cakici (2008) and Fu (2009) find a positive relationship between idiosyncratic volatility and returns, arguing the puzzle does not exist while Ang et al. (2006, 2009), Li et al. (2008) and Arena et al. (2008) find that the puzzle persists, reporting evidence of a negative relationship. Furthermore, a conditional idiosyncratic component of stock return volatility is found to be positively related to returns by Chua et al. (2010) and Fu (2009), while conflicting results are found in Li et al. (2008). Despite the use of a variety of theoretical models of agents' behaviour, pricing models and testing techniques, the debate is still open as to whether idiosyncratic risk is a relevant cross-sectional driver of return, and if it is, whether the relationship with returns is a positive or a negative one. The contribution of our paper may be viewed in this context as an attempt to shed further light on these open and persistent questions. There is also evidence that several additional cross-sectional risk factors interact with residual risk effects, such as momentum, size and liquidity suggesting that a large part of it might be

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[4] Jacobs and Wang (2004) develop a consumption-based asset pricing model in which expected returns are a function of cross-sectional (across individuals) average consumption growth and consumption dispersion (the cross-sectional variance of consumption growth). The model predicts (and the evidence supports) a higher expected return the more negatively correlated the stock's return is with consumption dispersion. An intuitive interpretation is that consumption dispersion causes agents to perceive their own individual risk to be higher. Hence a stock which is sensitive to consumption dispersion offers a hedge, will be willingly held and consequently has a lower required return. Stocks with high idiosyncratic volatilities may have high exposure to consumption dispersion, which lowers their average returns, indicating a negative pricing of idiosyncratic risk in the cross-section

systematic rather than idiosyncratic (Malkiel and Xu (1997, 2002), Campbell et al. (2001), Bekaert et al. (2012) and Ang et al. (2009)).

There is a problem when researchers test the CAPM empirically using *ex-post* realized returns in place of *ex-ante* expected returns, upon which the CAPM is based. When realized returns are used Pettengill et al. (1995) argue that a conditional relationship between beta and return should exist in the cross-section of stocks. In periods when the excess market returns is positive (negative) a positive (negative) relation between beta and returns should exist. Pettengill et al. (1995) propose a model with a conditional relationship between beta and return and find strong support for a systematic but conditional relationship. Lewellen and Nagel (2006) show, however, that the conditional CAPM is not a panacea and does not explain pricing anomalies like value and momentum.

The majority of empirical work deals with U.S. data. Morelli (2011) examines the conditional relationship between beta and returns in the UK market. The author highlights the importance of this conditionality for only then is beta found to be a significant risk factor. Given the evidence of a conditional beta/return relationship established in the literature, our paper makes a further contribution by incorporating this conditionality in re-examining the pricing of idiosyncratic risk. We focus on a UK dataset while obtaining results of general interest in terms of methodological approach and empirical results.

The paper is set out as follows: section 2 describes the selection and treatment of data while section 3 describes our testing methodology. Results are discussed in section 4 while Section 5 concludes.

2. Data Treatment and Selection

Our starting universe includes all stocks listed on the London Stock Exchange between January 1990 and December 2009 – a period long enough to capture economic cycles, latterly the ‘financial crisis’ and alternative risk regimes. We collect monthly prices, total returns, volume, outstanding shares and static classification information from Datastream. We also daily prices in order to compute quoted spread, a liquidity measure, as well as 1-month GBP Libor rates. Serious issues with international equity data have been highlighted in the literature (Ince and Porter, 2006). These include incorrect information, both qualitative (classification information) and quantitative (prices, returns, volume, shares etc), a lack of distinction between the various types of securities traded on equity exchanges, issues of coverage and survivorship bias, incorrect information on stock splits, closing prices and dividend payments, problems with total returns calculation and with the time markers for beginning and ending points of price data and with handling of returns after suspension periods. Ince and Porter (2006) also flag problems caused by rounding of stock prices and with small values of the return index. Most (not all) of the problems identified are concentrated in the smaller size deciles and this issue would significantly impact inferences drawn by studies focusing on cross-sectional stock characteristics. We thus apply great care to mitigate these problems by defining strict data quality filters to improve the reliability of price and volume data and to ensure results are economically meaningful for investors. First, we review all classification information with a mix of manual and automatic techniques, including a cross-check of all static information against a second data source, Bloomberg.[5]. Second, we cross-check all time-series information (prices, returns, shares, volume) against Bloomberg, correcting a large number of issues and recovering data for a significant number of constituents that were missing[6]. These data filters result in a comprehensive sample of 1,333 stocks. Full details of our data cleaning procedures are available on request.

[5] "Manual" means, in many cases, a name-by-name, ISIN-by-ISIN check of the data, or the retrieval and incorporation of data from company websites. As commonly done, in this first step we exclude (i) investment trusts and other types of non-common-stock instruments, eliminating securities not flagged as equity in Datastream, (ii) securities not denominated in GBP, (iii) unit trusts, investment trusts, preferred shares, American depositary receipts, warrants, split issues, (iv) securities without adjusted price history, (v) securities flagged as secondary listings for the company, (vi) stocks identified as non-UK under the Industry Classification Benchmark (ICB) system, (vii) securities without a minimum return history of 24 months and (viii) non-common stock constituents, mis-classified as common-stock, by searching for key words in their names - for instance, collective investment funds are have been identified and excluded.

[6] The error rate in Datastream and the much higher reliability of stock-level data in Bloomberg raises the question of why we do not simply use Bloomberg as our data source. There are various reasons including that only Datastream allows queries for bulk data with a common characteristic (i.e. all stocks listed on the London exchange) and licensing issues.

3. The Pricing of Idiosyncratic Risk: Theory and Empirical Methods

We use a two-step procedure similar to Fama and MacBeth (1973) to test for the pricing of cross-sectional risk factors[7]. In our first step we estimate a time series regression of the form (Fama and French, 1992)

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + h_i HML_t + s_i SMB_t + \varepsilon_{i,t}, \quad i = 1,2\ldots n \tag{1}$$

where $R_{i,t}$ is the excess return (over the risk free rate) on stock *i* at time *t*, $R_{m,t}$ is the excess return on the market portfolio, β_i represents the systematic risk of stock *i*, HML_t, the difference in returns between high versus low book to market equity stocks, is a value risk factor at time *t*, h_i is the value risk factor loading on stock *i*, SMB_t, the difference in returns between small versus big stocks, is a size risk factor at time *t* while s_i is the size risk factor loading on stock *i*[8]. $\varepsilon_{i,t}$ represents idiosyncratic variation in stock *i* and *n* is the number of stocks in the cross-section. (In some tests we examine the CAPM version of [1], i.e., without the value and size risk factors). We estimate [1] each month using a backward looking window of 24 months, rolling the window forward one month at a time[9]. We collect the series of $\hat{\beta}_i$, $\hat{h}_i$ and $\hat{s}_i$ each month and generate estimates of the idiosyncratic risk of stock *i*, denoted $\hat{\sigma}_i$, using the series of the residuals $\hat{\varepsilon}_{i,t}$ based on four alternative approaches as follows:

(i) the standard deviation of the series of $\hat{\varepsilon}_{i,t}$ over the 24 months rolling window,

[7] We provide only a brief outline of this well-known procedure here.

[8] The monthly returns for the HML factor are obtained from Kenneth French's website, available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, while we compute SMB by sorting stocks into size deciles based on market capitalization and taking the spread in return between the top and bottom decile portfolios.

[9] The data frequency, backward looking window length and forward rolling frequency vary in previous literature. For instance, Malkiel and Xu (2002) and Spiegel and Wang (2005) employ monthly data with a backward looking window of 60 months, Li et al.(2008) use windows of 3, 6 and 12 months, Hamao et al.(2003) use monthly data over a 12 month window. A number of studies such as Ang et al.(2009) and Bekaert et al.(2007) use daily data over one month. Brockman et al.(2009) use both daily data and monthly data. We use monthly data for consistency with our following cross-sectional analysis and a window length of 24 months as sufficiently long to ensure reliable risk estimators in each window but short enough to capture changing risk over time.

(ii) the fitted value at *t-1* from a GARCH(1,1) model fitted to the series of $\hat{\varepsilon}_{i,t}$ over the 24 months window,

(iii) generating each month a forecast of the conditional volatility of $\hat{\varepsilon}_{i,t}$ from a GARCH(1,1) model fitted over the 24 month window,

(iv) the fitted value from an EGARCH(1,1) model fitted to the series of $\hat{\varepsilon}_{i,t}$ over the 24 months window[10].

In the second stage, a cross-sectional regression is estimated each month of the form

$$R_{i,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{i,t-1} + \gamma_{2,t}\hat{\sigma}_{i,t-1} + u_{i,t} \tag{2}$$

where $u_{i,t}$ is a random error term. Subscript *t-1* denotes that $\hat{\beta}_i$ and $\hat{\sigma}_i$ are estimated in the 24 month window up to time *t-1*. It is advisable to obtain systematic and idiosyncratic risk estimates from [1] from month *t-1* through month *t-24* and then relate these to security returns in month *t* in [2] in order to mitigate the Miller-Scholes problem.[11] This procedure provides estimates $\hat{\gamma}_{0,t}$, $\hat{\gamma}_{1,t}$ and $\hat{\gamma}_{2,t}$ each month *t*. Under CAPM, H_{10}: $\hat{\gamma}_{0,t} = 0$, $H_{20} : \hat{\gamma}_{1,t} = R_{M,t}$ and H_{30}: $\hat{\gamma}_{2,t} = 0$. Under normally distributed i.i.d. returns, $t_{\hat{\gamma}_j} = \dfrac{\bar{\hat{\gamma}}_j}{\sigma_{\hat{\gamma}_j}}$, *j* = 0,1,2, is distributed as a student's t-distribution with T-1 degrees of freedom where T is the number of observations, $\bar{\hat{\gamma}}_j$ and $\sigma_{\hat{\gamma}_j}$ are the means and standard deviations respectively of the time series of the cross-sectional

[10] In cases (i) to (iv) for robustness we also run tests where idiosyncratic risk is estimated using a backward looking 12 month window instead of 24 month and report a selection of results in Section 4.

[11] Miller and Scholes (1972) find that individual security returns are marked by significant positive skewness so that firms with high average returns will typically have large measured total or residual variances as well. This suggests caution when using total or residual variance as an explanatory variable, as substantiated in practice by Fama and MacBeth (1973) who found total risk added to the explanatory power of systematic factor loadings in accounting for stock mean returns only when the same observations were used to estimate mean returns, factor loadings and total variances. Similar results were obtained by Roll and Ross (1980) in their tests of the Arbitrage Pricing Theory

coefficients estimated monthly. The CAPM asserts that systematic risk is positively priced and this may be tested empirically by $H_0 : \bar{\hat{\gamma}}_1 = 0$ versus $H_A : \bar{\hat{\gamma}}_1 > 0$.

However, there is a problem when researchers test the model empirically using *ex-post* realized returns rather than the *ex-ante* expected returns upon which the CAPM is based. When realized returns are used Pettengill et al. (1995) argue that a conditional relationship between beta and return should exist in the cross-section of stocks. This arises because the model implicitly assumes that there is some non-zero probability that the realized market return, $R_{m,t}$, will be less than the risk free rate, i.e., $R_{m,t} < R_f$ as well as some non-zero probability that the realized return of a low beta security will be greater than that of a high beta security[12].

Pettengill et al. (1995) propose a conditional relationship between beta and return of the form

$$R_{i,t} = \lambda_{0,t} + \lambda_{1,t} D\hat{\beta}_i + \lambda_{2,t}(1 - D)\hat{\beta}_i + \varepsilon_{i,t} \tag{3}$$

where $R_{i,t}$ is the realised excess return on stock *i* in month *t*, D is a dummy variable equal to one (zero) when the excess market return is positive (negative). Equation (3) is estimated each month. The model implies that either $\lambda_{1,t}$ or $\lambda_{2,t}$ will be estimated in a given month depending on whether the excess market return is positive or negative. The hypotheses to be tested are $H_{1,0} : \bar{\hat{\lambda}}_1 = 0, H_{1,A} : \bar{\hat{\lambda}}_1 > 0$ and $H_{2,0} : \bar{\hat{\lambda}}_2 = 0, H_{2,A} : \bar{\hat{\lambda}}_2 < 0$ where $\bar{\hat{\lambda}}_1$ and $\bar{\hat{\lambda}}_2$ are the time series averages of the cross-sectional coefficients estimated monthly. These hypotheses can be tested by the t-tests of Fama and MacBeth (1973). Our final testing model incorporating a conditional beta/return relationship, idiosyncratic risk and the rolling backward looking estimation window is of the following form,

[12] We provide a fuller review of the analytics of the conditional CAPM in an appendix to the paper.

$$R_{i,t} = \lambda_{0,t} + \lambda_{1,t} D\hat{\beta}_{i,t-1} + \lambda_{2,t}(1-D)\hat{\beta}_{i,t-1} + \lambda_{3,t}\hat{\sigma}_{i,t-1} + \varepsilon_{i,t} \quad (4)$$

where $\varepsilon_{i,t}$ is a random error term. The time series averages of the lambda coefficients are then calculated and statistical significance tested.

3.1 Additional Control Variables in the Cross-sectional Regressions.

A number of other cross-sectional variables have been shown to interact with residual risk and we attempt to control for these by augmenting [4]. Factors such as size, value, liquidity and momentum have been documented in the literature. Malkiel and Xu (1997) report evidence of a strong relationship between idiosyncratic volatility and size, suggesting that the two variables may be partly capturing the same underlying risk factors. Similar findings are reported in Malkiel and Xu (2002), Chua et al. (2010) and Fu (2009). Spiegel and Wang (2005) show that liquidity interacts strongly with idiosyncratic risk while a strong relationship between momentum returns and idiosyncratic volatility has been documented in Ang et al.(2006), Li et al. (2008) and Arena et al.(2008).

We augment [4] with the size and value risk factor loadings estimates from [1], again estimated between *t*-1 and *t*-24. For robustness we also examine the role of size as measured by a standardised measure of market capitalization at time *t*. The literature contains several alternative measures of liquidity, Foran et al. (2014a). We adopt two measures including the quoted spread and turnover, which have been found to explain the cross-section of UK equity returns, Foran et al. (2014b). The quoted spread is the difference between the closing bid and ask prices expressed as a percentage of the midpoint of the prices. We calculate the daily average each month. For month *m* and stock *s* it is given by

$$Q_{s,m} = \frac{1}{n_{s,m}} * \sum_{t=1}^{n_{s,m}} \frac{P_{s,t}^{A} - P_{s,t}^{B}}{m_{s,t}} \quad (5)$$

where $P^A_{s,t}$ is the ask price on day *t* for stock *s*, $P^B_{s,t}$ is the bid price on day *t* for stock *s*, $n_{s,m}$ is the number of daily observations in month *m* and $m_{s,t} = (P^A_{s,t} + P^B_{s,t})/2$ is the midpoint of the bid-ask prices. Higher levels of quoted spread are associated with lower levels of liquidity. Turnover is defined as the volume of shares traded per period divided by the total number of shares outstanding. Higher levels of turnover are associated with higher liquidity. As turnover varies over time at both the market-wide level and at stock level, we also decompose it into a systematic component and an idiosyncratic component. We decompose turnover by estimating a time series regression for each stock of the form

$$TURN_{i,t} = \varphi_0 + \varphi_1 TURN_{MKT,t} + \theta_{i,t} \quad (6)$$

over a 24 month backward looking window and rolling the window forward one month at a time as before. $TURN_{i,t}$ is the turnover of stock *i* at time *t*, $TURN_{MKT,t}$ is the market-cap weighted average of individual stocks' turnover at time *t*. While φ_1 measures the sensitivity of each stock's turnover to market-wide turnover, $\theta_{i,t}$ is a measure of turnover that is unique to each firm. We augment [4] at time *t* with $\hat{\varphi}_1$ estimated over *t-1* to *t-24* and with $\hat{\theta}_{i,t-1}$. We find for the most part that time-variation in stock turnover comes from the systematic component.

We measure momentum as the stock's cumulative return over the past 3 months. This is the measurement period that yields the most significant winners/losers spread in Li et al. (2008). Finally, two recent papers question the existence of the idiosyncratic risk pricing puzzle and propose additional control variables in testing its existence. Bali, Cakici and Whitelaw (2011) argue that including the maximum daily return over the previous month reverses the negative relationship while Huang et al. (2010) argue that the puzzle disappears on controlling for short run (one month) return reversal, i.e., the return at *t-1*, though this variable is likely to interact with momentum here. We further augment [4] with these additional control variables[13].

[13] We thank an anonymous referee for this suggestion.

In Table 1 we provide descriptive statistics of the stock returns, beta and idiosyncratic volatility while in Figure 1 we chart the cross-sectional average idiosyncratic volatility (averaged across stocks using market capitalisation weights) over time. For example, from Table 1, the time series and cross-sectional average stock return is 1.17% per month with a large standard deviation of 13.71%. The average market beta is 1.027 from the CAPM version of [1] (averaged over the rolling 24 month windows and across stocks), falling to 0.938 in the Fama and French (1992) model in [1]. The means of the idiosyncratic volatility measures are broadly similar ranging from 6.96% per month in the case of 'IVOL-FF-EGARCH', which denotes the value at *t-1* from an EGARCH(1,1) model fitted to the series of $\hat{\varepsilon}_{i,t}$ over the backward looking 24 months window from *t-1* to *t-24*, to 9.18% in the case of 'IVOL-CAPM', which denotes the standard deviation of residuals from a CAPM version of [1] estimated over the backward looking 24 month window. Figure 1 also reveals a similar trend in idiosyncratic volatility over time between the alternative measures, rising in the late 1990s around events such as the Russian debt default and Asian currency crises and rising again from 2008 during the more recent financial crisis.

[Table 1 about here]

[Figure 1 about here]

4. Empirical Results

We estimate the cross-sectional regressions in [4] each month *t*. These regressions examine the pricing of systematic risk, β, idiosyncratic risk, σ, as well as other risk factors including liquidity, value, size and momentum while also specifying some other control variables. As described in Section 3, $\hat{\beta}$ and $\hat{\sigma}$ are estimated over the previous 24 months (and also over the previous 12 months in robustness tests). We present results in Tables 2, 3 and 4. Initially, in Table 2 we estimate an unconditional cross-sectional regression each month over the entire sample period and ignore the possible conditional beta/return relationship. In Tables 3 and 4 we estimate various forms of [4] which models the beta/return relation as conditional: Table 3 reports results for down-markets while Table 4 presents results for up-markets[14]. We build an array of models, gradually introducing cross-sectional factors and robustness tests. For each model we report the time series averages of the coefficients from the monthly cross-sectional regressions with their p values below. In all our time series regression in [1] as well as our cross-sectional regressions in [4] all standard errors are Newey-West (1987) adjusted (lag order 2).

Across all three tables, models 1-8 report results for monthly cross-sectional regressions of stock returns on a constant, market risk (denoted 'beta') and alternative estimates of idiosyncratic risk as follows: (i) 'IVOL-FF' denotes the standard deviation of residuals from the Fama and French (1992) model in [1] estimated over a backward looking 24 month window from *t-1* to *t-24*, while 'IVOL-CAPM' is similarly estimated but built on the CAPM version of [1], i.e., without the value and size risk factors. 'IVOL-FF-12m' is estimated similarly to 'IVOL-FF' except it is based on a backward looking window of 12 months. 'IVOL-FF-GARCH' denotes the fitted value at *t-1* from a GARCH(1,1) model fitted to the series of $\hat{\varepsilon}_{i,t}$ over the 24 months window from [1], while 'IVOL-CAPM-GARCH' is estimated similarly from the residuals of the CAPM version of [1]. 'F-IVOL-FF-GARCH' is obtained by fitting a GARCH(1,1) to the variance of the residuals in [1] over a 24 month backward looking window and generating each month a forecast of the conditional volatility, while 'F-IVOL-CAPM-GARCH' is estimated similarly based on the residuals from the CAPM version of [1]. Finally, 'IVOL-FF-EGARCH' denotes the value at *t-1* from an EGARCH(1,1) model fitted to the series of $\hat{\varepsilon}_{i,t}$ over the backward looking 24 months window.

In model 9 through 25 we introduce the other risk factors and control variables in the cross-sectional regressions and report robustness test results around idiosyncratic risk measures.

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[14] In [4] we estimate: $R_{i,t} = \lambda_{0,t} + \lambda_{1,t} D\hat{\beta}_{i,t-1} + \lambda_{2,t}(1-D)\hat{\beta}_{i,t-1} + \lambda_{3,t}\hat{\sigma}_{i,t-1} + \varepsilon_{i,t}$. Here only the $R_{i,t} / \hat{\beta}_{i,t-1}$ is conditional. The $R_{i,t} / \hat{\sigma}_{i,t-1}$ relation is unconditional. However, in effect, testing the conditional $R_{i,t} / \hat{\beta}_{i,t-1}$ relation involves estimating it in down-markets and up-markets separately. Similarly, it may be enlightening to examine a conditional $R_{i,t} / \hat{\sigma}_{i,t-1}$ relation and indeed a conditional relationship between return and the other risk including value, size, liquidity and momentum. Our results in Table 2 are based on unconditional tests while Tables 3 and 4 report results for down-markets and up-markets respectively.

‘TURN’, ‘Beta-TURN’, and ‘I-TURN’ denote total turnover, systematic turnover and idiosyncratic turnover respectively while Quoted Spread is also specified as a further measure of liquidity. ‘H’ and ‘S’ represent the value and size risk factor loadings from [1] again estimated over a backward looking window of 24 months, while ‘Mkt Val std’ denotes a stock’s standardised market capitalisation in month *t*. A momentum factor, denoted ‘mom 3m’ is also specified to allow for momentum effects in performance, this is the stock’s cumulative return over the past 3 months. Finally, as described in Section 3, two additional control variables are specified, i.e., ‘Return Reversal’ and ‘Max Daily Return’ which have been found to be relevant in the literature. Bali et al. (2011), Huang et al. (2010).

Our results across Tables 2 to 4 point to a number of striking findings. First, there is strong evidence of a conditional beta/return relationship as predicted and found by Pettengill et al. (1995). Under the column denoted ‘Beta’ we observe from Table 3 (down-markets) a negative beta/return relation which is consistently statistically significant at the 1% significance level across all models. From Table 4 (up-markets) we observe a positive beta/return relation which, again, is consistently statistically significant at the 1% significance level across all models. This finding is strongly robust to the specification in the cross-sectional regressions of the alternative estimates of idiosyncratic risk as well other risk and control variables. In results not tabulated, the coefficients on beta risk in down-markets versus up-markets are significantly different from each other at 5% significance. In Table 2, which combines down-markets and up-markets in unconditional tests, we see that the beta/return relation varies from positive to negative and is not significant at 5% significance – this is a feature of the averaging over the up and down-market cycles and disguises the beta/return conditionality.

A second striking finding across Tables 2 to 4 is that our results support (i) a conditional relationship between idiosyncratic risk and return and (ii) the idiosyncratic risk puzzle, i.e., that idiosyncratic risk is negatively priced in the cross-section of stocks. From Table 3 (down-markets), the relation is negative and statistically significant at the 5% significance level in all models and for all measures of idiosyncratic risk. However, in Table 4 (up-markets), the relation is positive in all models, except one, but statistically insignificant in all models, except two. It is statistically insignificant for all measures of idiosyncratic risk except IVOL-GARCH and IVOL-FF-EGARCH. The coefficients on idiosyncratic risk in down-markets versus up-markets are significantly different from each other at 5% significance (results not tabulated for brevity). In the unconditional test results in Table 2, the relation between idiosyncratic risk and return is negative across all models but predominantly statistically insignificant. These two findings around the pricing of beta risk and idiosyncratic risk are key contributions of our paper and underline the importance of modelling the beta/return as conditional.

[Table 1, Table 2 and Table 3 about here]

Among the additional cross-sectional and control variables, our results indicate that turnover as a measure of liquidity is, counter-intuitively, positively priced in stock returns. However, this finding holds in the unconditional full sample and in up-markets but not in down-markets. The unusual positive pricing of liquidity in UK stock returns is consistent with past findings among (unconditional) studies of the UK market, Foran et al. (2014b), Lu and Hwang (2007) and may arise because of an interaction between liquidity and momentum risk: our unconditional test results in Table 2 suggest that turnover (liquidity) and momentum represent distinct effects where they are both statistically significant variables. However, this is not the case in the conditional test results in Tables 3 and 4. Foran et al. (2014c) also report evidence of an interaction between liquidity and momentum risks. When we decompose turnover into a systematic and idiosyncratic component, however, we find that neither is statistically significant in the cross-sectional regressions. We reach a similar conclusion regarding the quoted spread measure of liquidity. Our conditional testing approach also reveals a mixed effect for size risk on stock returns: in the combined sample of down-markets and up-markets, the size risk factor loading is not a significant determinant of returns but in up-markets alone it is positive and significantly priced in all models tested. We find no evidence for the pricing of value risk. Finally, our max daily return control variable, (highest value of daily return over the past month) is not statistically significant while the return reversal variable is unexpectedly positively signed (and generally statistically significant), suggesting it may be picking up a momentum effect rather than a return reversal effect.

In summary, we find strong evidence in support of the Pettengill et al. (1995) argument that the relationship between stock returns and beta is conditional on whether the excess return in the market is positive or negative. Furthermore, we confirm the findings of Morelli (2011) who finds that only under this conditionality is beta found to be a significant determinant of stock returns. Critically, our results also point to conditionality in the pricing of idiosyncratic risk and uphold the idiosyncratic risk puzzle. Although perhaps initially counter-intuitive, this finding is consistent with the theory put forward by Chen (2002) and Ang et al (2006) as outlined in Section 2 which predicts that idiosyncratic volatility risk is negatively priced due to its link with market volatility risk. The Chen (2002) model predicts a negative loading on the covariance between a stock's return and changes in the forecasts of future market volatilities indicating that investors demand compensation in the form of higher expected return for the risk that an asset will perform poorly when the future becomes less certain. Ang et al (2006) argue that stocks with high idiosyncratic volatilities may particularly exhibit this characteristic. Our results strongly indicate that this negative pricing effect is further accentuated in down-markets when investors need to pursue high levels of diversification to offset it.

5. Conclusion

Using a large and long sample of UK stock returns we re-examine the role of idiosyncratic risk in asset pricing. A distinguishing feature of our approach is that we allow for a conditional relationship between beta risk and returns in our tests. We find strong evidence for this conditional beta/ return relationship. In unconditional tests, the beta/return relation is not significant. The conditional testing framework also reveals a conditional relationship between idiosyncratic risk and returns where, in addition, the idiosyncratic risk puzzle is upheld, i.e., a negative idiosyncratic risk/return relation. This negative relation exists in down market cycles – a highly significant findings which is robust to alternative measures of idiosyncratic risk and several model specifications which allow for additional risk factors and control variables. Our findings support some extant theories that predict that idiosyncratic volatility risk is negatively priced due to its link with market volatility risk. In the case of size and liquidity risk exposures, our results again suggest that pricing is conditional on up-markets versus down-markets although we leave a fuller investigation of this to future research.

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Table 1. Descriptive statistics for systematic and idiosyncratic risk - pooled sample.

Variables	**Mean**	**Median**	**Standard Deviation**
Return	1.17%	0.80%	13.71%
Beta	1.027	0.939	1.102
Beta FF	0.938	0.885	0.919
IVOL-CAPM	9.18%	7.75%	5.73%
IVOL-CAPM-GARCH	7.62%	6.55%	4.70%
F-IVOL-CAPM-GARCH	7.71%	6.58%	4.88%
IVOL-FF	8.39%	7.08%	5.50%
IVOL-FF-12m	8.36%	6.89%	6.02%
IVOL-FF-GARCH	8.72%	7.12%	6.69%
IVOL-FF-EGARCH	6.96%	5.12%	9.24%
F-IVOL-FF-GARCH	8.82%	7.14%	6.93%

The Table 1 shows descriptive statistics for beta and idiosyncratic volatility pooled across stocks and over time.

Table 2. Regressions of returns on cross-sectional stock characteristics: unconditional tests.

Model	Constant	Beta	IVOL-CAPM	IVOL-CAPM-GARCH	F-IVOL-CAPM-GARCH	IVOL-FF	IVOL-FF-12m	IVOL-FF-GARCH	IVOL-FF-EGARCH	F-IVOL-FF GARCH	TURN	Beta-TURN	I-TURN	Quoted Spread	H	S	Mkt Val std	Mom 3m	Return Reversal	Max Daliy Return
1	0.01	-0.002	-0.01																	
	0.00	0.21	0.46																	
2	0.01	-0.001		-0.02																
	0.00	0.26		0.20																
3	0.008	-0.001			-0.02															
	0.00	0.26			0.20															
4	0.008	-0.002				-0.02														
	0.01	0.16				0.44														
5	0.008	-0.002					-0.02													
	0.00	0.15					0.32													
6	0.01	0.00						-0.03												
	0.00	0.24						0.14												
7	0.01	0.00							-0.01											
	0.01	0.17							0.47											
8	0.008	-0.001								-0.02										
	0.00	0.22								0.21										
9	0.01	0.00	-0.01								0.01									
	0.01	0.19	0.49								0.00									
10	0.01	0.00	-0.01									0.00								
	0.00	0.10	0.80									0.23								
11	0.01	0.00	-0.02										0.01							
	0.00	0.19	0.44										0.54							
12	0.01	0.00		-0.02										0.01						
	0.00	0.08		0.25										0.76						
12	0.01	-0.002				-0.02					0.01				0.00	0.00				
	0.01	0.14				0.42					0.00				0.19	0.51				
13	0.01	-0.002						-0.027			0.01				0.00	0.00				
	0.00	0.20						0.10			0.00				0.18	0.35				
14	0.01	-0.001						-0.022			0.01				0.00		0.00			
	0.01	0.29						0.21			0.00				0.27		0.31			
15	0.01	0.00						-0.03			0.01				0.00					
	0.00	0.34						0.14			0.00				0.27					
16	0.01	0.00						-0.03			0.01				0.00				0.01	
	0.01	0.94						0.09			0.00				0.04				0.08	
17	0.01	0.00						-0.03			0.01				0.00			0.02		
	0.01	0.27						0.09			0.01				0.24			0.00		
18	0.01	0.00						-0.03			0.01				0.00			0.02		0.02
	0.01	0.19						0.04			0.02				0.24			0.00		0.20
19	0.01	0.00						-0.03			0.01				0.00	0.00		0.02		0.02
	0.01	0.11						0.03			0.02				0.26	0.37		0.00		0.16
20	0.01	0.00							-0.01		0.01				0.00	0.00		0.02		0.02
	0.03	0.08							0.16		0.02				0.21	0.64		0.00		0.21
21	0.01	0.00				-0.02					0.01				0.00	0.00		0.02		0.02
	0.02	0.07				0.26					0.02				0.27	0.50		0.00		0.25
22	0.01	0.00			-0.03						0.01				0.00	0.00		0.02		0.03
	0.01	0.11			0.02						0.02				0.29	0.39		0.00		0.09
23	0.01	0.00		-0.03							0.01				0.00	0.00		0.02		0.02
	0.01	0.10		0.03							0.02				0.31	0.36		0.00		0.16
24	0.01	0.00	-0.02								0.01				0.00	0.00		0.02		0.02
	0.01	0.09	0.15								0.02				0.29	0.43		0.00		0.23

Table 2 shows the results of our two-step asset pricing tests. In the first step, each month for each stock we run a time series regression of stock returns on market, size and value risk factors over the previous 24 months to estimate risk factor loadings. We estimate alternative measures of idiosyncratic risk from the residuals of this regression. In the second step we regress stock returns on beta and idiosyncratic risk as well as on factors for liquidity, value, size and momentum risk as well as other control variables in a cross-sectional regression. We roll this two-step procedure forward one month at a time. Full details of the 24 models are outlined in the text. For each model we report the time series average of the coefficients in the monthly cross-sectional regressions with p-values below.

Table 3. Regressions of returns on cross-sectional stock characteristics: Down-markets.

Model	Constant	Beta	IVOL-CAPM	IVOL-CAPM-GARCH	F-IVOL-CAPM-GARCH	IVOL-FF	IVOL-FF-12m	IVOL-FF-GARCH	IVOL-FF-EGARCH	F-IVOL-FF GARCH	TURN	Beta-TURN	I-TURN	Quoted Spread	H	S	Mkt Val std	Mom 3m	Return Reversal	Max Daliy Return
1	-0.02	-0.012	-0.09																	
	0.00	0.00	0.01																	
2	-0.02	-0.012		-0.10																
	0.00	0.00		0.00																
3	-0.018	-0.012			-0.10															
	0.00	0.00			0.00															
4	-0.022	-0.010				-0.11														
	0.00	0.00				0.00														
5	-0.021	-0.010					-0.12													
	0.00	0.00					0.00													
6	-0.02	-0.01						-0.12												
	0.00	0.00						0.00												
7	-0.02	-0.01							-0.08											
	0.00	0.00							0.00											
8	-0.021	-0.010								-0.11										
	0.00	0.00								0.00										
9	-0.02	-0.01	-0.09								0.01									
	0.00	0.00	0.01								0.36									
10	-0.02	-0.01	-0.08									0.00								
	0.00	0.00	0.01									0.07								
11	-0.02	-0.01	-0.09										0.01							
	0.00	0.00	0.01										0.44							
12	-0.02	-0.01		-0.12										-0.01						
	0.00	0.00		0.00										0.86						
12	-0.02	-0.012				-0.07					0.01				0.00	0.00				
	0.00	0.00				0.03					0.40				0.69	0.02				
13	-0.02	-0.012						-0.098			0.00				0.00	0.00				
	0.00	0.00						0.00			0.40				0.63	0.07				
14	-0.02	-0.011						-0.101			0.01				0.00		0.00			
	0.00	0.00						0.00			0.34				0.62		0.46			
15	-0.02	-0.01						-0.10			0.01				0.00					
	0.00	0.00						0.00			0.35				0.61					
16	-0.02	-0.01						-0.11			0.01				0.00				0.05	
	0.00	0.00						0.00			0.38				0.41				0.00	
17	-0.02	-0.01						-0.10			0.00				0.00			0.04		
	0.00	0.00						0.00			0.47				0.94			0.00		
18	-0.02	-0.01						-0.10			0.00				0.00			0.04		-0.01
	0.00	0.00						0.00			0.76				0.88			0.00		0.66
19	-0.02	-0.01						-0.09			0.00				0.00	0.00		0.04		-0.01
	0.00	0.00						0.00			0.80				0.95	0.07		0.00		0.71
20	-0.02	-0.01							-0.06		0.00				0.00	0.00		0.04		-0.01
	0.00	0.00							0.00		0.73				0.90	0.03		0.00		0.59
21	-0.02	-0.01				-0.06					0.00				0.00	0.00		0.04		-0.02
	0.00	0.00				0.04					0.76				0.99	0.03		0.00		0.42
22	-0.02	-0.01			-0.10						0.00				0.00	0.00		0.04		0.00
	0.00	0.00			0.00						0.75				0.96	0.14		0.00		0.81
23	-0.02	-0.01		-0.10							0.00				0.00	0.00		0.04		0.00
	0.00	0.00		0.00							0.79				0.93	0.14		0.00		0.86
24	-0.02	-0.01	-0.07								0.00				0.00	0.00		0.04		-0.01
	0.00	0.00	0.01								0.76				0.97	0.06		0.00		0.53

Table 3 shows the results of our two-step asset pricing tests in down-markets only. In the first step, each month for each stock we run a time series regression of stock returns on market, size and value risk factors over the previous 24 months to estimate risk factor loadings. We estimate alternative measures of idiosyncratic risk from the residuals of this regression. In the second step we regress stock returns on beta and idiosyncratic risk as well as on factors for liquidity, value, size and momentum risk as well as other control variables in a cross-sectional regression. We roll this two-step procedure forward one month at a time. Full details of the 24 models are outlined in the text. For each model we report the time series average of the coefficients in the monthly cross-sectional regressions with p-values below.

Table 4. Regressions of returns on cross-sectional stock characteristics: Up-markets.

Model	Constant	Beta	IVOL-CAPM	IVOL-CAPM-GARCH	F-IVOL-CAPM-GARCH	IVOL-FF	IVOL-FF-12m	IVOL-FF-GARCH	IVOL-FF-EGARCH	F-IVOL-FF GARCH	TURN	Beta-TURN	I-TURN	Quoted Spread	H	S	Mkt Val std	Mom 3m	Return Reversal	Max Daliy Return
1	0.03	0.005	0.03																	
	0.00	0.00	0.22																	
2	0.03	0.005		0.03																
	0.00	0.00		0.17																
3	0.025	0.005			0.03															
	0.00	0.00			0.13															
4	0.026	0.004				0.04														
	0.00	0.00				0.14														
5	0.026	0.004					0.04													
	0.00	0.00					0.10													
6	0.03	0.00						0.03												
	0.00	0.00						0.15												
7	0.03	0.004							0.04											
	0.00	0.00							0.02											
8	0.027	0.004								0.03										
	0.00	0.00								0.14										
9	0.02	0.00	0.03								0.01									
	0.00	0.00	0.22								0.00									
10	0.03	0.00	0.04									0.00								
	0.00	0.01	0.10									0.90								
11	0.03	0.00	0.03										0.00							
	0.00	0.00	0.24										0.88							
12	0.03	0.00		0.05										0.02						
	0.00	0.01		0.03										0.60						
12	0.03	0.005				0.02					0.01				0.00	0.00				
	0.00	0.00				0.49					0.00				0.16	0.00				
13	0.03	0.005						0.017			0.01				0.00	0.00				
	0.00	0.00						0.43			0.00				0.17	0.00				
14	0.03	0.005						0.028			0.01				0.00		0.00			
	0.00	0.00						0.21			0.00				0.31		0.04			
15	0.03	0.00						0.02			0.01				0.00					
	0.00	0.00						0.26			0.00				0.31					
16	0.03	0.01						-0.01			0.02				0.00				0.00	
	0.00	0.00						0.59			0.33				0.05				0.01	
17	0.03	0.00						0.02			0.01				0.00			0.01		
	0.00	0.00						0.38			0.00				0.13			0.22		
18	0.03	0.00						0.01			0.01				0.00			0.01		0.04
	0.00	0.00						0.63			0.00				0.15			0.28		0.07
19	0.02	0.00						0.00			0.01				0.00	0.00		0.01		0.04
	0.00	0.01						0.84			0.00				0.13	0.01		0.20		0.06
20	0.02	0.00							0.02		0.01				0.00	0.00		0.01		0.04
	0.00	0.00							0.20		0.00				0.09	0.01		0.14		0.07
21	0.02	0.00				0.01					0.01				0.00	0.00		0.01		0.04
	0.00	0.01				0.81					0.00				0.11	0.01		0.17		0.07
22	0.02	0.00			0.01						0.01				0.00	0.00		0.01		0.04
	0.00	0.01			0.60						0.00				0.14	0.01		0.17		0.07
23	0.02	0.00		0.01							0.01				0.00	0.00		0.01		0.04
	0.00	0.01		0.55							0.00				0.18	0.01		0.19		0.08
24	0.02	0.00	0.01								0.01				0.00	0.00		0.01		0.04
	0.00	0.01	0.70								0.00				0.12	0.01		0.17		0.08

Table 4 shows the results of our two-step asset pricing tests in up-markets only. In the first step, each month for each stock we run a time series regression of stock returns on market, size and value risk factors over the previous 24 months to estimate risk factor loadings. We estimate alternative measures of idiosyncratic risk from the residuals of this regression. In the second step we regress stock returns on beta and idiosyncratic risk as well as on factors for liquidity, value, size and momentum risk as well as other control variables in a cross-sectional regression. We roll this two-step procedure forward one month at a time.

Full details of the 24 models are outlined in the text. For each model we report the time series average of the coefficients in the monthly cross-sectional regressions with p-values below.

Figure 1. Average idiosyncratic volatility, aggregated across stocks using a market-cap weighted average

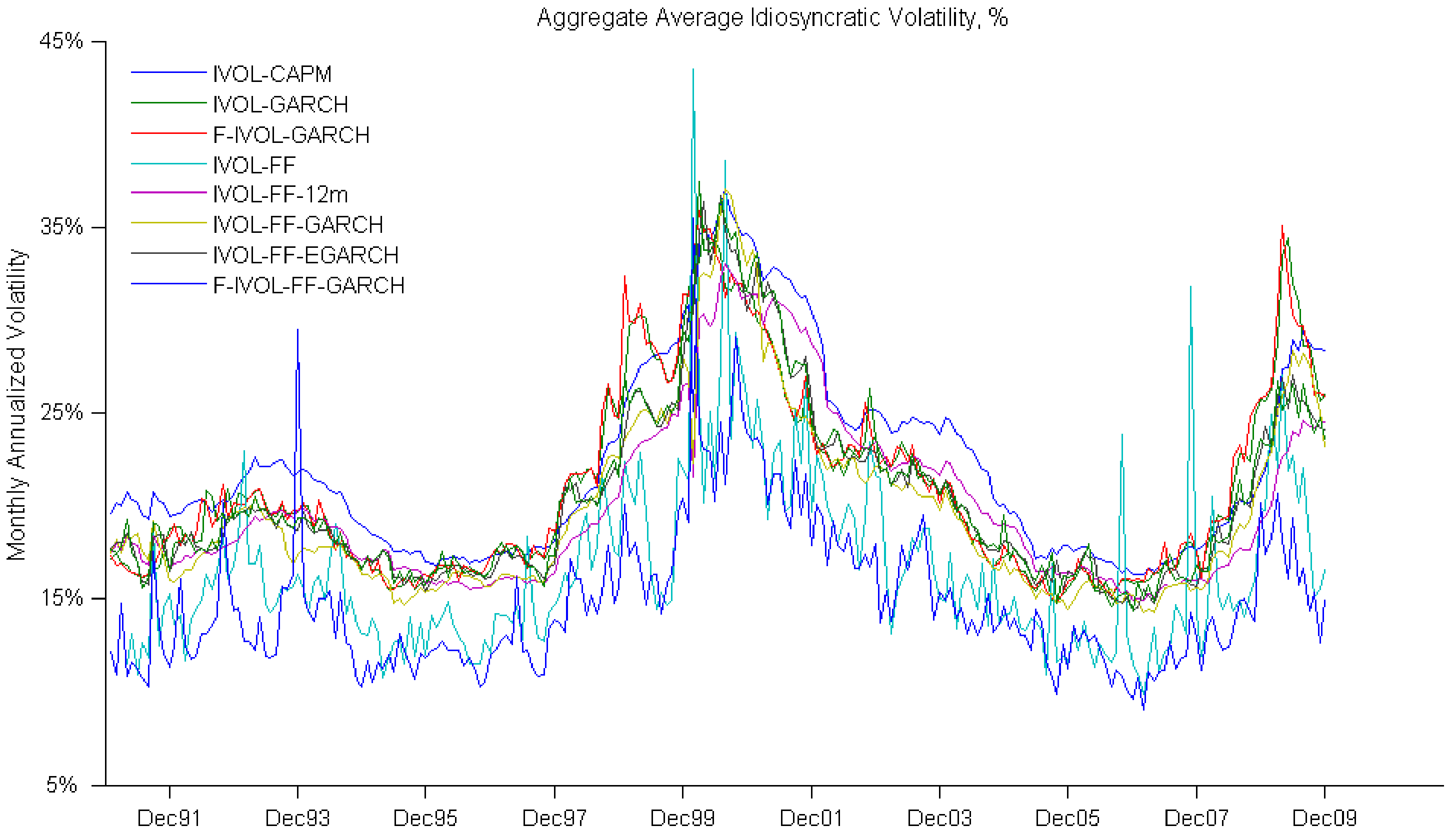

Figure 1 plots value-weighted averages of the alternative idiosyncratic volatility measures overtime.

APPENDIX

For a point in time, *t*, the realized market return comes from a distribution of possible returns. Similarly, the realized return on a security *i* comes from a distribution of possible returns. The CAPM asserts that the mean or expected values of these distributions are related as follows:

$$E(R_{i,t}) = R_f + \beta_{i,t}[E(R_{m,t}) - R_f)] \qquad \text{A1}$$

where $E(R_{i,t})$ is the expected return on security *i* at time *t*. R_f is the known return on a risk free asset over time *t*, $\beta_{i,t}$ is the security beta at time *t* and $E(R_{m,t})$ is the expected market return at time *t*. The model implicitly assumes that $E(R_{m,t}) > R_f$ as otherwise all investors would hold the risk free asset. Therefore, the model implies that in the cross-section of security returns $E(R_{i,t})$ is a positive function of $\beta_{i,t}$. There is a problem, however, when researchers test the model using realized returns instead of expected returns. This arises because the model also implicitly assumes that there is some non-zero probability that $R_{m,t} < R_f$, where $R_{m,t}$ is the realized market return as otherwise no investor would hold the risk free asset. The CAPM itself does not describe a relationship between $R_{i,t}$ and $\beta_{i,t}$ when $R_{m,t} < R_f$ as it does the positive relationship between $E(R_{i,t})$ and $\beta_{i,t}$. A further implication of the CAPM is that while a high beta security has a higher expected return than a low beta security to compensate for higher systematic risk, there must be some non-zero probability that the *realized* return of the low beta security will be greater than that of the high beta security as otherwise no investor would hold the low beta security. Pettengill et al (1995) suggest a reasonable inference is that this realization occurs when $R_{m,t} < R_f$. The implication of this is that there should be a positive (negative) relationship between beta and realized return when the excess market return is positive (negative). While the CAPM does not imply this relationship, the relationship is consistent with the market model, Jensen et al. (1972). This proposes

$$R_{i,t} = E(R_{i,t}) + U(R_{i,t}) \qquad \text{A2}$$

where the realized (excess) return on security *i* is the sum of an expected component and an unexpected component, $U(R_{i,t})$. A key assumption is the unexpected component is linearly related, through $\beta_{i,t}$, to the unexpected market (excess) return $R_{m,t} - E(R_{m,t})$ as follows

$$U(R_{i,t}) = \beta_{i,t}[R_{m,t} - E(R_{m,t})] + \varepsilon_{i,t} \qquad \text{A3}$$

where $R_{m,t} - E(R_{m,t})$ and $\varepsilon_{i,t}$ are normally distributed, uncorrelated, zero-mean random variables. By substitution this gives

$$R_{i,t} = E(R_{i,t}) + \beta_{i,t}[R_{m,t} - E(R_{m,t})] + \varepsilon_{i,t} \qquad \text{A4}$$

By the CAPM, $E(R_{i,t}) = \beta_{i,t}\, E(R_{m,t})$ and by further substitution

$$\begin{aligned} R_{i,t} &= \beta_{i,t}E(R_{m,t}) + \beta_{i,t}[R_{m,t} - E(R_{m,t})] + \varepsilon_{i,t} \\ R_{i,t} &= \beta_{i,t}R_{m,t} + \varepsilon_{i,t} \end{aligned} \qquad \text{A5}$$

This formulation implies a positive (negative) relationship between beta and realized return when the excess market return is positive (negative).